Scattering matrix representation for the incidence of electromagnetic waves on multiconductor transmission line structures.

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Abstract - In the present contribution, we demonstrate how circuits described by a coupled transmission line model with distributed sources can be represented by a classical S-matrix description. This approach allows one to use commercially available time and/or frequency domain simulators to study the circuit behaviour.

GEOMETRY AND FORMULATION OF THE PROBLEM

In [1, 2, 3, 4] amongst others circuit models are derived to account for the influence of an incident field impinging on a multiconductor transmission line system (MTL). This MTL with constant cross-section consists of N conductors. In many papers, the conductors are assumed to be cables above earth and located in free space ([1], [2]), in other contributions, the conductors may be embedded in a planar stratified medium ([3], [4]). In both cases, the coupled transmission line model one obtains to account for the influence of the incident field has the following form (the z-direction equals the propagation direction):

\[
\begin{align*}
\frac{dV(z,\omega)}{dz} + Z_0(\omega)I(z,\omega) &= V_g(z,\omega) \\
\frac{dI(z,\omega)}{dz} + Y(\omega)V(z,\omega) &= I_g(z,\omega)
\end{align*}
\]  

(1)

\(V(z,\omega), I(z,\omega), V_g(z,\omega), I_g(z,\omega)\) : vectors of N elements,
\(Z(\omega), Y(\omega)\) :
N x N impedance (admittance) matrix.
\(V_g(z,\omega)\) and \(I_g(z,\omega)\) are frequency and z-dependent, distributed sources. As most commercially available circuit simulators do not allow transmission line circuits with distributed sources, one is obliged to develop a new simulator or one has to look for an alternative formulation. In the next section (1) will be transformed into a description that can be handled by modern circuit simulators.

SCATTERING MATRIX DESCRIPTION OF THE PROBLEM

It is generally known that a circuit with active components can be described by an S-matrix formalism in the following way:

\[
B(\omega) = S(\omega)A(\omega) + C(\omega).
\]  

(2)

\(A(\omega), B(\omega), C(\omega)\) : vectors of N elements,
\(S(\omega) : (N x N) - matrix\).
\(S(\omega)\) is the global scattering matrix and \(C(\omega)\) the external source vector. \(S(\omega), A(\omega), B(\omega)\) and \(C(\omega)\) may be frequency dependent.

In the following the voltage wave vectors of the structure, described by equation (2), are defined with respect to the characteristic impedance matrix \(Z_C\) of the structure. \(A^1(\omega)\) and \(B^1(\omega)\) (\(A^2(\omega)\) and \(B^2(\omega)\)) represent the incident and reflected voltage wave vectors at the beginning (end) of the conductors (see Fig. 1). As the conductors may be coupled, the voltages and impedances in Fig. 1 at the line ends are represented by matrices. With these definitions, one obtains the following S-matrix description for the MTL structure under study (The superscript indicates the port number, the subscript gives the dimension of the matrix; \(d\) is the length of the considered MTL-
section and $\Delta_v$ is the complex voltage propagation matrix of the MTL.

$$\begin{pmatrix}
B_1(\omega)_{N \times 1} \\
B_2(\omega)_{N \times 1}
\end{pmatrix} = B_{2N \times 1}$$

$$= \begin{pmatrix}
\frac{1}{Z_c e^{-\Delta_v d}} e^{-\frac{1}{2} \Delta_v d} \\
\frac{1}{Z_c e^{-\Delta_v d}} e^{-\frac{1}{2} \Delta_v d}
\end{pmatrix} \begin{pmatrix}
A_1(\omega) \\
A_2(\omega)
\end{pmatrix}$$

$$+ \begin{pmatrix}
-\frac{1}{2} Z_c e^{\Delta_v d} \\
\frac{1}{2} Z_c e^{-\Delta_v d}
\end{pmatrix} \begin{pmatrix}
\int e^{-\Delta_v z} (V_{g - Z_c I_g}) dz \\
\int e^{\Delta_v z} (V_{g + Z_c I_g}) dz
\end{pmatrix}$$

$$= S_{2N \times 2N} A_{2N \times 1} + C_{2N \times 1}$$

![Fig. 1: A 2N coupled port system characterised by a $S$- and $C$-matrix.](image1)

This means that when the influence of an incident field on a MTL with $N$ conductors is studied the active part of the structure is represented by a $[2N \times 1]$-$C$-matrix while the passive part is described by the $[2N \times 2N]$-$S$-matrix. This system can be transformed to an equivalent network, described by an active $[4N \times 4N]$-$S$-matrix and with $2N$ additional external sources:

$$\begin{pmatrix}
B_{2N \times 1} \\
C_{2N \times 1}
\end{pmatrix} = \begin{pmatrix}
S_{2N \times 2N} & 0_{2N \times 2N} \\
0_{2N \times 2N} & I_{2N \times 2N}
\end{pmatrix} \begin{pmatrix}
A_{2N \times 1} \\
C_{2N \times 1}
\end{pmatrix}$$  (4)

where $I_{2N \times 2N}$ is the $2N \times 2N$ unitary matrix.

To obtain this, $2N$ additional ports are added to the circuit (Fig. 2). Each of these new ports is driven by a voltage source, whose variation as a function of frequency is given by one of the elements of the $C$-matrix (one port for each element) multiplied by 2 times the square root of the reference impedance of the port. The impedance at each coupled port can be chosen freely. With this transformation, the internal active part $C$ (i.e. the sources $V_g$ and $I_g$) are represented as completely outside sources. This approach makes it possible to use widely available simulators such as HP-MDS and Libra to characterise the structures described by equation (1).

![Fig. 2: Active 4N coupled port system characterised by a $S$-matrix.](image2)

**NUMERICAL RESULTS**

Consider the configuration shown in Fig. 3. Two asymmetric strips are placed on a lossy substrate, they are 15 mm long and are terminated as shown in Fig. 4. The voltage source generates a sinusoidal wave of 10GHz with an amplitude of 2V. This structure is illuminated by a horizontal electrical dipole $I_0 = I_{0x} \delta(x) \delta(y-2mm)$ with $J=1mA mm$, situated at 2 mm above the ground plane.

![Fig. 3: Cross-section of the structure under study.](image3)

Fig. 5 shows the induced voltage at the beginning and at the end of the left and of the right conductor respectively, in the presence of the electrical dipole. In Fig. 6 the dipole is removed. One notices that only in node 1 the incident field couples destructively, in all the other nodes, the voltage increases due to the presence of the interfering source.
CONCLUSION

A method to transform a circuit model with distributed voltage and current sources into a classical \( S \)-matrix description is demonstrated. It is shown that in this way it is possible to use commonly available circuit simulators to study the circuit behaviour.

REFERENCES


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Fig. 4: Line terminations of the structure under study.

Fig. 5: Voltage at the line ends, in the presence of the dipole.

Fig. 6: Voltage at the line ends, in the absence of the dipole.