Abstract—This paper presents a fast iterative algorithm for passivity enforcement of large nonpassive macromodels that share a common set of poles. It is ensured that the maximum passivity violation is monotonically decreasing in each iteration step, and convergence to a passive macromodel is guaranteed.

Index Terms—Broadband macromodeling, numerical techniques, passivity enforcement, vector fitting.

I. INTRODUCTION

The synthesis of accurate broadband macromodels from tabulated $S$-parameter data is very important for the design of passive microwave systems and devices. Although standard identification techniques are available to extract the model coefficients with a high accuracy, the resulting macromodel is stable, but possibly nonpassive [1]–[3]. Nevertheless, passivity of the macromodel is of crucial importance since a nonpassive model may lead to unstable transient simulations in an unpredictable manner. Several techniques have been considered to address this issue, ranging from convex optimization [4] to Nevanlinna-pick interpolation [5], semidefinite programming [6], linear or quadratic programming [7], residue perturbation [8]–[10], pole perturbation [11], modal perturbation [12], waveform shaping [13], and others [14]–[17].

This paper introduces a new iterative algorithm that is able to enforce passivity by means of a fast pole perturbation scheme. By perturbing only the poles of the macromodel, it is possible to deal with large multiport systems that share a common set of poles in a very efficient way. Although the idea of pole perturbation has been considered before [11], the proposed method is substantially different. This approach perturbs the poles of the model while preserving the zeros, whereas [11] perturbs the poles of the model while preserving the residues. By considering the pole-zero form instead of the pole-residue form, it is possible to derive some analytic conditions which guarantee that the maximum passivity violation is monotonically decreasing in each iteration step. Therefore, it is also guaranteed that the proposed method will converge to a passive macromodel, as illustrated by three examples.

II. PASSIVITY CONDITION

The proposed method in this paper considers a stable, but potentially nonpassive multipport system in state-space form

$$\begin{align*}
  j\omega X(j\omega) &= AX(j\omega) + BU(j\omega) \\
  Y(j\omega) &= CX(j\omega) + DU(j\omega)
\end{align*}$$

provided that $A$ is the state matrix, $B$ is the input matrix, $C$ is the output matrix, and $D$ is the feedthrough matrix with appropriate dimensions [18]. A stable realization can be obtained by applying the fast vector fitting procedure to some tabulated $S$-parameter data, while enforcing a common pole set for each matrix element [19]. The associated transfer matrix $H(j\omega)$ is

$$H(j\omega) = C(j\omega I - A)^{-1}B + D,$$

In the case of scattering parameters, the exact definition of passivity stipulates that $H(j\omega)$ must be unitary bounded

$$I - H^*(j\omega)H(j\omega) \geq 0 \quad \forall \omega$$

such that the following equivalent condition is satisfied:

$$\max_{\omega}(\sigma(j\omega)) \leq 1 \quad \forall \sigma(H(j\omega)).$$

The singular values curves $\sigma(H(j\omega))$ are then defined as

$$\sigma(H(j\omega)) = \sqrt{\text{eig}(H^*(j\omega)H(j\omega))},$$

III. ANALYTIC PASSIVITY TEST

The passivity of the state-space model can easily be verified by computing the eigenvalues of a Hamiltonian matrix $M$ [21]

$$M = \begin{bmatrix} A - BR^{-1}D^T & -BR^{-1}B^T \\ C^TQ^{-1}C & -A^T + C^TDR^{-1}B^T \end{bmatrix}$$

where $R = D^TD-I$ and $Q = DD^T-I$. If $j\omega_k$ is an imaginary eigenvalue of the matrix $M$, then the corresponding frequency $\omega_k$ may denote the crossover between a passive and a nonpassive frequency band [20]. By computing the slopes of the singular value curves at the purely imaginary eigenvalues, it is possible to determine the exact boundaries of a passivity violation. If all the eigenvalues of the Hamiltonian matrix $M$ have a nonvanishing real part, then the system is passive [21].

IV. PERTURBATION CONSTRAINTS

If the macromodel is nonpassive, then the passivity enforcement algorithm perturbs the common poles of the state-space model until all singular value curves (5) are unitary bounded. During this process, it is ensured that the perturbation does not

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introduce new violations, which may result at other frequencies. Therefore, some additional constraints must be imposed, which guarantee that the size of the largest passivity violation decreases monotonically in each iteration step. Two specific cases are distinguished: the perturbation of a real pole and the perturbation of a complex conjugate pair of poles.

A. Constraints on a Real Pole

In the case of a real pole perturbation, the perturbed transfer matrix \( \tilde{H}(j\omega) \) is obtained by multiplying each matrix element of \( H(j\omega) \) by a frequency-dependent factor \( \alpha_r(j\omega) \). This factor cancels out the original real pole \( \alpha_p \) and introduces a perturbed real pole \( \tilde{\alpha}_p \), which yields a modified transfer function

\[
\tilde{H}(j\omega) = H(j\omega)\alpha_r(j\omega) = H(j\omega)(j\omega - \alpha_p) / (j\omega - \tilde{\alpha}_p). \tag{8}
\]

The singular values of \( \tilde{H}(j\omega) \) are then described as follows:

\[
\sigma(\tilde{H}(j\omega)) = \sqrt{\text{eig}(\alpha_r(j\omega)H^*(j\omega)H(j\omega)\alpha_r(j\omega))} = |\alpha_r(j\omega)|\sigma(H(j\omega)). \tag{9}
\]

To ensure that the compensation does not introduce new violations elsewhere, \( |\alpha_r(j\omega)| \leq 1 \) must hold for all \( \omega \), hence,

\[
\left| \frac{j\omega - \alpha_p}{j\omega - \tilde{\alpha}_p} \right| \leq 1 \Leftrightarrow |j\omega - \alpha_p|^2 \leq |j\omega - \tilde{\alpha}_p|^2 \quad \forall \omega \in \mathbb{R}. \tag{11}
\]

Since \( \tilde{\alpha}_p \) must be stable, it follows that \( \Re(\tilde{\alpha}_p) \leq \Re(\alpha_p) \).

B. Constraints on a Complex Pole Pair

In the case of a complex pole perturbation, the perturbed transfer matrix \( \tilde{H}(j\omega) \) is obtained by multiplying each matrix element of \( H(j\omega) \) by a frequency-dependent factor \( \alpha_c(j\omega) \). This factor cancels out the original complex conjugate poles \( \alpha_p, \alpha_p^* \) and introduces a perturbed set of complex conjugate poles \( \tilde{\alpha}_p, \tilde{\alpha}_p^* \), which yields a modified transfer function:

\[
\tilde{H}(j\omega) = H(j\omega)\alpha_c(j\omega) = H(j\omega)(j\omega - \alpha_p)(j\omega - \alpha_p^*) / (j\omega - \tilde{\alpha}_p)(j\omega - \tilde{\alpha}_p^*). \tag{12}
\]

The effect on the singular values of \( \tilde{H}(j\omega) \) is similar

\[
\sigma(\tilde{H}(j\omega)) = \sqrt{\text{eig}(\alpha_c(j\omega)H^*(j\omega)H(j\omega)\alpha_c(j\omega))} = |\alpha_c(j\omega)|\sigma(H(j\omega)). \tag{13}
\]

To ensure that the compensation does not introduce new violations elsewhere, \( |\alpha_c(j\omega)| \leq 1 \) must hold for all \( \omega \), hence,

\[
\left| \frac{(j\omega - \alpha_p)(j\omega - \alpha_p^*)}{(j\omega - \tilde{\alpha}_p)(j\omega - \tilde{\alpha}_p^*)} \right| \leq 1 \quad \forall \omega \in \mathbb{R} \tag{15}
\]

\[
\Leftrightarrow |j\omega - \alpha_p|^2 - |j\omega - \tilde{\alpha}_p|^2 \geq 0 \quad \forall \omega \in \mathbb{R} \tag{16}
\]

\[
\Leftrightarrow \omega^2(-2|\alpha_p|^2 + 4\Re(\tilde{\alpha}_p)^2 + 2|\alpha_p|^2 - 4\Re(\alpha_p)^2) + |\tilde{\alpha}_p|^4 - |\alpha_p|^4 \geq 0 \quad \forall \omega \in \mathbb{R} \tag{17}
\]

where the second-order polynomial on the left-hand side is non-negative for every \( \omega \in \mathbb{R} \) if and only if

\[
\Re(\alpha_p)^2 + 3\Im(\alpha_p)^2 \geq \Re(\tilde{\alpha}_p)^2 + 3\Im(\tilde{\alpha}_p)^2 \tag{18}
\]

and

\[
\Re(\tilde{\alpha}_p)^2 - 3\Im(\tilde{\alpha}_p)^2 \geq \Re(\alpha_p)^2 - 3\Im(\alpha_p)^2 \tag{19}
\]

provided that the perturbed complex poles \( \tilde{\alpha}_p, \tilde{\alpha}_p^* \) are stable.

C. Visualization of Perturbation Area

The previous conditions allow the algorithm to pinpoint exactly the region in which a perturbed pole can be located without introducing new passivity violations. It suffices to add some attenuation to real poles, but more relaxed conditions are derived for complex conjugate poles (18), (19). In the latter case, the complex pole perturbation area is bounded by the following.

- A circle (18) that is centered at the origin with radius \( |\alpha_p| \), having the following parametric coordinates

\[
\Re(\tilde{\alpha}_p) = |\alpha_p| \cos(t) \quad \Im(\tilde{\alpha}_p) = |\alpha_p| \sin(t). \tag{20}
\]

- An orthogonal hyperbola (19) that is centered at the origin with a given parameter \( \gamma = \Re(\alpha_p)^2 - 3\Im(\alpha_p)^2 \).

- If \( \gamma > 0 \), then the hyperbola has an east–west opening

\[
\Re(\tilde{\alpha}_p) = \sqrt{\gamma} \sec(t) \quad \Im(\tilde{\alpha}_p) = \sqrt{\gamma} \tan(t). \tag{21}
\]

- If \( \gamma < 0 \), then the hyperbola has a north–south opening

\[
\Re(\tilde{\alpha}_p) = \sqrt{\gamma} \tan(t) \quad \Im(\tilde{\alpha}_p) = \sqrt{\gamma} \sec(t). \tag{22}
\]

- If \( \gamma = 0 \), then the hyperbola reduces to the asymptotes.

A visual illustration of the circle and hyperbola is shown in Fig. 1 (\( \gamma > 0 \)), Fig. 2 (\( \gamma < 0 \)), and Fig. 3 (\( \gamma = 0 \)), where the valid pole perturbation area is marked in grey.

V. Perturbation Algorithm

This section presents a simple, but efficient scheme that removes passivity violations by perturbing the poles of the macromodel. First, the Hamiltonian passivity check (as described in Section III) is used to determine the passivity of the macromodel. If the macromodel is found to be nonpassive, then an iterative algorithm is applied, which consists of the following steps.

Step 1) Find the frequency \( \omega_{\text{viol}} \) that corresponds to the largest passivity violation based on the eigenvalues of (7).

Step 2) Select the pole \( \alpha_p \) of the model for which the contribution to the largest passivity violation \( \Delta H_p(\omega_{\text{viol}}) \) is maximal.

Step 3) Perturb the pole such that the passivity violation becomes smaller, without introducing new violations. At the same time, minimize the model deviation by error control.

These steps are repeated iteratively until the macromodel is assumed passive. As a final step, the passivity is verified by re-computing the eigenvalues of a Hamiltonian matrix.
A. Selection of the Relevant Pole

As mentioned in Section III, it is possible to exactly pinpoint the nonpassive regions of the spectrum by considering the purely imaginary eigenvalues of the Hamiltonian matrix (7). A simple optimization algorithm can then be applied to find the frequency $\omega_{\text{viol}}$, which corresponds to the largest passivity violation. This problem converges fast since the optimization involves only one variable and the midpoints of each interval can be used as a good initial guess. Details about this procedure are well described in the literature (see [8] and [22]).

For each pole $a_p$ of the macromodel, the contribution $\Delta H_p(\omega_{\text{viol}})$ to the largest passivity violation can be computed as the $L_2$-norm of the corresponding residue matrix fraction

$$
\Delta H_p(\omega_{\text{viol}}) = \left\| \frac{C_p}{j\omega_{\text{viol}} - a_p} \right\|_2, \quad \text{for } p = 1, \ldots, P.
$$

The pole $a_p$ that corresponds to the largest contribution $\Delta H_p(\omega_{\text{viol}})$ is perturbed to compensate the passivity violation.

B. Perturbation of the Pole

If $a_p$ is a real pole, then it suffices to add some attenuation such that $a_p = ra_p$, where $r > 1$ is a positive real parameter.

If $a_p$ is a complex pole, then a circle is formed, which is centered at the pole $a_p$, having a small positive radius $r > 0$

$$(x - \Re(a_p))^2 + (y - \Im(a_p))^2 = r^2,$$

Fig. 3. Pole perturbation area for $\{a_p, a_p^*\} = -2 \pm 2j (\gamma = 0)$.

Fig. 4. Second quadrant of Fig. 2: dots represent valid candidate poles.
Some tuples $(x_k, y_k)$, which are equidistantly spread over the circle (24), are chosen to form complex conjugate pairs of candidate poles $\tilde{p}_{\mu,k} = x_k + jy_k$ and $\tilde{p}_{\nu,k}^* = x_k - jy_k$. Only the valid pole pairs, which satisfy the perturbation constraints (18) and (19), are retained. For each of the remaining pole pairs, the algorithm computes the least squares error $\|H(j\omega) - H(j\omega^*)\|_2$ that would be introduced over the frequency range of interest if the original poles are replaced by the candidate poles [see (12)]. The pole pair that corresponds to the smallest error is then effectively used for the replacement. A visual illustration is shown in Fig. 4 in case $\gamma < 0$. Although a minimization of the least squares fitting error is suggested in this paper, it is clear that any kind of error criterion (absolute or relative) can be used instead [11], [23].

C. Iteration Scheme

This perturbation process is repeated iteratively until all passivity violations are removed. Since the perturbation of the poles guarantees that no new passivity violations are introduced in each iteration step, it suffices to compute the eigenvalues of the Hamiltonian matrix only once at the beginning and once at the end of the algorithm. It is noted that the convergence speed of the iteration scheme depends on the value of $r$ that is chosen in Section V-B. In practice, it can be chosen in such a way that the pole perturbation factor $\tilde{\alpha}_p(j\omega)$ and $\tilde{\alpha}_c(j\omega)$ in (10) or (14) compensates approximately a certain percentage of the largest passivity violation in each iteration step. The percentage is a tuning parameter that allows the designer to find an acceptable tradeoff between efficiency (a few large compensations) and accuracy (several small compensations).

VI. EXAMPLE: 48-PORT BALL GRID ARRAY (BGA) PACKAGE

In this example, the presented approach is used to compute a passive macromodel of a 48-port BGA package [11]. The scattering parameters of the structure are simulated with Agilent EEsof Momentum [24] from dc up to 10 GHz, and vector fitting is used to approximate the response by a six-pole proper transfer function using 100 data samples [7]. It is seen from Fig. 5 that the macromodel has several nonnegligible passivity
violations both inside and outside the frequency range of interest. The proposed passivity enforcement procedure is applied to compensate the violations, and converges to a passive macromodel in only 18 s on a Dual Core 2.4-GHz laptop computer. Figs. 6 and 7 show that the accuracy of the overall macromodel is well preserved, both in terms of the magnitude and the phase angle, respectively. The worst case error over all matrix elements is $-43$ dB, which is quite small given the size of the maximum violation ($\sigma_{\text{max}} = 1.0009$). The rms deviation that was introduced by the perturbations corresponds to $3 \times 10^{-2}$. It is seen from Fig. 8 that the maximum singular value of the scattering matrix decreases monotonically in each iteration step. The algorithm converges in ten iterations to a guaranteed passive macromodel.

VII. EXAMPLE: TWO-PORT HAIRPIN FILTER

In this example, the presented approach is used to compute a passive macromodel of a two-port microwave hairpin filter [11]. The scattering parameters of the structure are simulated in the frequency domain and vector fitting is used to approximate the frequency response by a ten-pole proper transfer function [7]. It is seen from Fig. 9 that the macromodel has some in-band passivity violations at the higher frequencies. The passivity enforcement procedure is applied to compensate them, and converges to a passive macromodel in only 2.8 s on the same laptop computer. Figs. 10 and 11 show that the accuracy of the overall macromodel is again well preserved, both in terms of the magnitude and the phase angle respectively. The rms deviation that was introduced by the algorithm from dc up to 15 GHz corresponds to $8 \times 10^{-3}$, which is acceptable. As can be seen from Fig. 12, the maximum singular value of the scattering matrix decreases monotonically in each iteration step, and the algorithm converges in 20 iterations to a guaranteed overall passive macromodel.
VIII. EXAMPLE: FOUR-PORT INTERCONNECT

In this example, the presented approach is used to compute a passive macromodel of a four-port interconnect system [25]. The scattering parameters of the structure are measured in the frequency domain from 0.775 to 7.52 GHz and vector fitting is used to approximate the response by a 100-pole proper transfer function [7]. It is seen from Fig. 13 that the macromodel has quite a large outband passivity violation at the lower frequencies. The passivity enforcement procedure is applied to compensate them, and converges to a passive macromodel in only 1.8 s. The rms deviation between the original and the passive model is $3 \times 10^{-2}$. Due to the size of the passivity violation, some visible difference can be distinguished between the frequency response of both models in the vicinity of the violation, as shown in Figs. 14 and 15. It is clear from Fig. 16 that the maximum singular value of the scattering matrix decreases monotonically in each iteration step, and that the algorithm converges in four iterations to a guaranteed passive macromodel.

IX. CONCLUSION

This paper has presented an iterative algorithm for passivity enforcement of large state space macromodels, which are based on a common pole set. The maximum singular value of the scattering matrix decreased monotonically in each iteration step, and convergence to a passive macromodel is guaranteed. Three examples have illustrated the efficiency of the approach.

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REFERENCES


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