Selection of Lumped Element Models for Coupled Lossy Transmission Lines

Tom Dhaene and Daniël De Zutter

***Abstract*—A practical method is developed for selecting the minimal number of lumped elements needed to represent a lossy transmission line if a certain accuracy is desired in a well-defined frequency range. This method, which uses dimensionless transmission line parameters, can be used in a wide range of applications and is also extended to hybrid equivalent circuits, consisting of ideal single lossless lines and resistors. For completeness, a new lumped element model for coupled lossy lines is presented which uses the same dimensionless parameters and the same criteria as proposed for single lines. An example of a coupled transmission line structure including skin-effect losses illustrates the approach.**

I. Introduction

Transient analysis of coupled lossy transmission lines with nonlinear loads is very important for the study and the design of high-speed electronic circuits. Owing to the increasing frequencies and bit rates, one has to take the transmission line effects of the interconnections into account, for example, signal-delay, reflection, attenuation, and cross talk. The performance of high-speed systems is mainly determined and limited by the ability to transmit signals undisturbed, undistorted, and with the desired speed. Quite often, the interconnections are terminated with nonlinear loads such as diodes or transistors. These nonlinear terminations, together with the coupled lossy transmission lines, constitute a rather complicated simulation problem.

There are really only two basic ways to handle transmission lines in transient-response simulation: by convolving with their impulse responses or using lumped element equivalent circuits.

The first approach is based on Green's functions. The basic procedure is always quite similar. First, the dispersive transmission line structures are evaluated in the frequency domain. Next, the frequency-domain data are transformed into the time-domain Green's functions. Then, the problem is solved in a time-marching fashion: after each time step, relevant transmission line parameters are computed by convolving the Green's functions with previous voltage and current values. The distinguishing feature at the different methods [1]–[4] is the choice of the Green's functions. Although these methods give good results, they require long simulation times and large memory space, and they are often not implemented in or compatible with popular general-purpose circuit simulators, such as ASTAP or SPICE.

The second approach is easier to implement. Lossy transmission lines can be modeled by lumped elements (e.g., T cells) or by a combination of lumped elements and ideal lossless transmission lines (hybrid cells). Lumped element models are quite often used in the literature [5]–[16], but the number of basic cells needed to accurately model a transmission line in a desired frequency range is seldom discussed. The number of cells must be large enough for accuracy reasons, but also small enough to avoid complicated models and large simulation times. Usually, a simple rule of thumb [10], [13] is used to select the number of cells: *the propagation delay caused by an elementary cell should be smaller than one fifth the shortest rise time expected.* The goal of this study is to formulate a methodology for selecting the number of cells if a certain accuracy is desired. The major advantage of the lumped element approach is that the equivalent circuits can be easily and quickly implemented in existing circuit simulators and all available facilities of these programs such as models for linear and nonlinear components and extensive input–output facilities can be used. Normally, the size of the circuit is not a problem.

We do not suggest that the lumped element approach is superior to other existing approaches. We only propose to investigate its validity and accuracy, allowing potential users to better discriminate those cases where the application of lumped elements is still appropriate from those where the other techniques are imperative.

First, three relevant dimensionless transmission line parameters are defined. Then frequency-domain error criteria are proposed and evaluated. For completeness, this approach is further extended to coupled lossy lines, and finally some examples are presented.

II. Transmission Line Model

A. Normalizations

In the quasi-TEM approximation, the propagation of a sinusoidal signal along a transmission line depends upon

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The authors are with the Laboratory of Electromagnetism and Acoustics (LEA), University of Ghent, Sint-Pietersnieuwstraat 41,9000 Ghent, Belgium.

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six important parameters:

$$\begin{array}{lll} d & [\mathrm{m}] & : \text{length of the line} \\ f & [1/\mathrm{s}] & : \text{frequency} \\ C & [\mathrm{s}/(\Omega \cdot \mathrm{m})] & : \text{capacitance per unit length} \\ L & [\mathrm{s} \cdot \Omega/\mathrm{m}] & : \text{inductance per unit length} \\ R & [\Omega/\mathrm{m}] & : \text{resistance per unit length} \\ G & [1/(\Omega \cdot \mathrm{m})] & : \text{conductance per unit length.} \end{array} \tag{1}$$

Each arbitrary time signal can be seen as a sum of sines.

It is difficult to evaluate a model as a function of these six parameters. To reduce the number of relevant parameters we make use of the Π-theorem of Vashy–Buckingham [17], [18], which states that if there are m independent parameters and n main dimensions, then it is always possible to write the dependent parameters as a function of $(m - n)$ dimensionless independent groups. These groups (Π-groups) are products of the independent parameters.

A transmission line has six independent parameters and three main dimensions (s, Ω, m); thus three relevant independent dimensionless Π-groups can be chosen.

1) The Normalized Frequency, f_N:

$$f_N = fd\sqrt{LC} = \frac{d}{\lambda} = f\tau_D \tag{2}$$

where

$$\lambda = \frac{1}{f\sqrt{LC}} \tag{3}$$

and

$$\tau_D = d\sqrt{LC}. \tag{4}$$

The parameter f_N is equal to the length of the line, d, measured in wavelengths, λ, of the corresponding lossless transmission line with the same inductance and capacitance per unit length. The normalized frequency, f_N, can also be seen as the product of the frequency, f, and the time delay, τ_D, of the corresponding lossless line. Usually, f_N is a rather small parameter (< 1).

2) The Normalized Resistance, R_N:

$$R_N = \frac{dR}{\sqrt{\frac{L}{C}}}. \tag{5}$$

The parameter R_N is equal to the ratio of the total dc resistance of the line to the characteristic impedance of the corresponding lossless line. This gives an indication of the relative resistance.

3) The Normalized Conductance, G_N:

$$G_N = dG\sqrt{\frac{L}{C}}. \tag{6}$$

The parameter G_N is equal to the total dc conductance of the line multiplied by the characteristic impedance of the corresponding lossless line. This gives an indication of the relative conductance.

From now on, all dependent parameters will be expressed as a function of the normalized parameters f_N, R_N, and G_N. These universal parameters allow us to use the results of this paper in a wide range of applications, such as cable distribution, long-distance power lines, and short, high-speed electronic interconnections, although the frequency range of interest may be completely different.

B. Lumped Element Models

A lossy transmission line can be represented as a cascade of an infinite number of infinitesimally small elementary *RGLC* cells. A lumped element equivalent circuit consists of a finite number of cells, N, large enough to meet the accuracy requirements and small enough to limit calculation time. Different elementary cells are used, such as Γ, T, and Π cells (Fig. 1). The accuracy of the different types of cells is quite similar. In this paper the symmetric T cell is described.

A lossy transmission line and a lumped element model are both linear, stationary, passive, reciprocal two-ports. In the frequency domain, different matrix representations are used. For example, the ***ABCD*** matrix of a single transmission line, with length d, is given by

$$\boldsymbol{ABCD} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} = \begin{bmatrix} \cosh \gamma d & Z_c \sinh \gamma d \\ \frac{\sinh \gamma d}{Z_c} & \coth \gamma d \end{bmatrix} \tag{7}$$

where the propagation factor, γ, and the characteristic impedance, Z_c, are defined as

$$\begin{aligned} \gamma d &= \sqrt{(R + sL)(G + sC)}\, d \\ &= \sqrt{(R_N + s_N)(G_N + s_N)} \end{aligned}$$

$$Z_c = \sqrt{\frac{R + sL}{G + sC}} = \sqrt{\frac{L}{C}}\sqrt{\frac{R_N + s_N}{G_N + s_N}}. \tag{8}$$

The normalized Laplace operator is given by

$$s_N = s\,\sqrt{LC}\,d = (\sigma + j2\pi f)\,\sqrt{LC}\,d = \sigma_N + j2\pi f_N. \tag{9}$$

Note that the ***ABCD*** transfer functions are *not rational* owing to the distributed nature of the transmission line.

The ***ABCD*** matrix of a T cell is given by

$$\begin{bmatrix} A_{\mathrm{Tcell}} & B_{\mathrm{Tcell}} \\ C_{\mathrm{Tcell}} & D_{\mathrm{Tcell}} \end{bmatrix} = \begin{bmatrix} \gamma^2 \frac{d^2}{2N^2} + 1 & Z_c\left(\gamma^3 \frac{d^3}{4N^3} + \gamma \frac{d}{N}\right) \\ \frac{\gamma d}{Z_c N} & \gamma^2 \frac{d^2}{2N^2} + 1 \end{bmatrix} \tag{10}$$

The transfer functions are *rational functions* and they have a finite number of zeros. Remark that $A = D$ and that

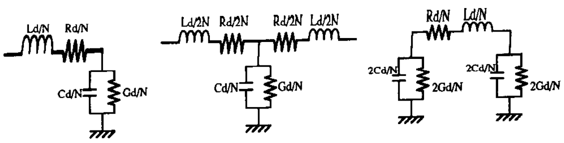


Fig. 1. Γ, T, and Π cells.

det$(\boldsymbol{ABCD}) = 1$ due to the symmetry and the reciprocity of the T cell.

The voltage and the current of a transmission line are functions of space (z) and time (t). A lumped element model approximates the voltage and the current at a finite number of points along the line as a function of time (discretization of the space parameter). Increasing the number of cells makes the model more accurate, but also more complex. In the literature, the number of lumped element cells required is rarely discussed. Quite often a simple rule of thumb is used [10], [13].

C. Accuracy of Lumped Element Models

The accuracy of a lumped element model depends on the line parameters (R, G, L, C),the line length (d), the number of cells (N), and the selected frequency range (f). To evaluate the accuracy of a model, several error criteria may be used. In this paper we look to the relative errors on the propagation factor, γ, and the characteristic impedance, Z_c, on the natural frequencies of the line, and on the $\boldsymbol{ABCD}$ matrix coefficients. Each error criterion evaluates a specific characteristic of the transmission line. The use of one or more criteria depends on the importance of certain transmission line properties and on the specific application.

1) Relative Error on Propagation Factor and Characteristic Impedance: Each two-port is characterized by a characteristic impedance and a propagation factor [8]. Errors on the characteristic impedance can cause wrong reflections and errors on the propagation factor cause incorrect propagation behavior.

The characteristic impedance of the symmetric T cell is given by

$$Z_c^{\mathrm{T\,cell}} = \sqrt{\frac{B_{\mathrm{T\,cell}}}{C_{\mathrm{T\,cell}}}} = Z_c \sqrt{1 + \gamma^2 \frac{d^2}{4N^2}} \tag{11}$$

where $B_{\mathrm{T\,cell}}$ and $C_{\mathrm{T\,cell}}$ are defined in (10). It can be shown by induction that a cascade of T cells has the same characteristic impedance, $Z_c^{\mathrm{T\,cell}}$. Note that $Z_c^{\mathrm{T\,cell}}$ approaches Z_c as N becomes larger.

The propagation factor, $\gamma_{\mathrm{T\,cell}}$, of one T-cell is given by

$$\gamma_{\mathrm{T\,cell}} = \frac{N}{d} \cosh^{-1} A_{\mathrm{T\,cell}} \tag{12}$$

where $A_{\mathrm{T\,cell}}$ is defined in (8).

Equations (10) and (12) lead to the following relation between the propagation factors of the lumped element model and the transmission line:

$$\gamma = \gamma_{\mathrm{T\,cell}} \sqrt{1 + \frac{2}{4!}\left(\gamma_{\mathrm{T\,cell}} \frac{d}{N}\right)^2 + \frac{2}{6!}\left(\gamma_{\mathrm{T\,cell}} \frac{d}{N}\right)^4 + \cdots}\,. \tag{13}$$

Note that $\gamma_{\mathrm{T\,cell}}$ approaches γ as N becomes larger.

We define the relative error on the characteristic impedance as

$$\epsilon_{Z_c} = \left|\frac{Z_c - Z_c^{\mathrm{T\,cell}}}{Z_c}\right| = \left|1 - \sqrt{1 + \left(\gamma \frac{d}{2N}\right)^2}\right|. \tag{14}$$

This relative error is shown in parts (a) and (b) of Fig. 2 as a function of the normalized frequency, f_N, for different numbers of cells and for different parameters R_N and G_N. These figures can be used to select the number of cells needed. The relative error increases if N decreases or if f_N increases. Note the dc error if $(R_N \cdot G_N) > 0$, which decreases if N increases.

In the lossless case, (14) reduces to

$$N = \frac{\pi f_N}{\sqrt{\epsilon_{Z_c}(2 - \epsilon_{Z_c})}}, \qquad f_N \leq N/\pi. \tag{15}$$

If an error of 2.5% on Z_c is accepted, this becomes

$$N \geq 14.14\, fd\sqrt{LC} \tag{16}$$

or, using the simple formula $Bt_{r10\%\text{-}90\%} = 0.35$, which relates the rise time of a signal to its bandwidth, B, one gets

$$N \geq 5 \frac{d\sqrt{LC}}{t_{r10\%\text{-}90\%}}. \tag{17}$$

A frequently used rule of thumb is here theoretically derived: *the propagation delay caused by an elementary cell should never be larger than one fifth the shortest rise time expected* [10], [13]. In the lossless case, this rule guarantees a maximum error, ϵ_{Z_c}, of 2.5% on the characteristic impedance and it ensures good accuracy for reflection and transmission behavior. A comparison of parts (a) and (b) of Fig. 2 shows that this rule can also be used for modeling lossy lines.

The relative error on the propagation factor, ϵ_γ, is less important since a hyperbolic function of γ appears in all relevant transfer functions. Note that in the lossless case ($R_N = G_N = 0$), ϵ_γ can be seen as the relative error on the delay time.

2) Relative Error on Natural Frequencies: Any transfer function is determined by the location of its poles and zeros [5]. The natural frequencies of a loaded line are given by the imaginary part of the poles of the input impedance. These poles or natural frequencies determine the transient and the dynamic behavior. So, it is important to examine the relative error on the natural frequencies for evaluating the accuracy of a lumped element model.

The input impedance, Z_{in}, of a transmission line loaded

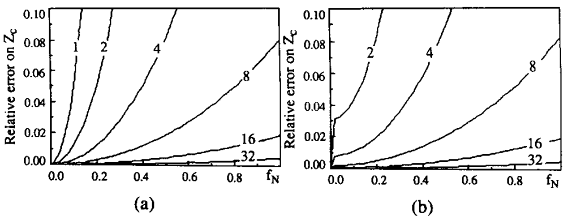


Fig. 2. Relative error on the characteristic impedance for different numbers of cells: (a) $R_N = G_N = 0$; (b) $R_N = G_N = 1$.

with Z_L is given by

$$Z_{\text{in}} = Z_c \frac{Z_L \cosh \gamma d + Z_c \sinh \gamma d}{Z_L \sinh \gamma d + Z_c \cosh \gamma d}. \tag{18}$$

Two important special cases are considered. The input impedances of an open- and a short-circuited transmission line, corresponding to the extreme loading conditions $Z_L = \infty$ and $Z_L = 0$, are directly found from (18):

$$Z_{\text{in}}(Z_L = \infty) = Z_O = Z_c \frac{\sinh \gamma d}{\cosh \gamma d}. \tag{19}$$

and

$$Z_{\text{in}}(Z_L = 0) = Z_S = Z_c \frac{\sinh \gamma d}{\cosh \gamma d}. \tag{20}$$

The (normalized) poles $s_{N_{\text{open}}}$ of (19) and $s_{N_{\text{short}}}$ of (20) are found to be

$$s_{N_{\text{open}}} = -\left(\frac{R_N + G_N}{2}\right) \pm \sqrt{\left(\frac{R_N - G_N}{2}\right)^2 - (m\pi)^2} \tag{21}$$

and

$$s_{N_{\text{short}}} = -\left(\frac{R_N + G_N}{2}\right) \pm \sqrt{\left(\frac{R_N - G_N}{2}\right)^2 - \left[(2m + 1)\frac{\pi}{2}\right]^2} \tag{22}$$

with $m = 0, 1, \cdots$. The poles and zeros of Z_O/Z_c correspond respectively to the zeros and poles of Z_S/Z_c.

The same quantities for the lumped element representation are

$$s_{N_{\text{open}}}^{\text{Tcell}} = -\left(\frac{R_N + G_N}{2}\right) \pm \sqrt{\left(\frac{R_N - G_N}{2}\right)^2 - \left(2N \sin \frac{m\pi}{2N}\right)^2} \tag{23}$$

and

$$s_{N_{\text{short}}}^{\text{Tcell}} = -\left(\frac{R_N + G_N}{2}\right) \pm \sqrt{\left(\frac{R_N - G_N}{2}\right)^2 - \left[2N \sin \left(\frac{2m - 1}{2N} \frac{\pi}{2}\right)\right]^2} \tag{24}$$

with $-(N - 1) \leq m \leq N$. The number of poles is limited by the discrete character of the model and the rational form of the transfer functions. However, a transmission line with distributed parameters has an infinite number of poles.

A lumped element model can only be used in the frequency range where the poles and the natural frequencies are accurately modeled. In the lossless case ($R_N = G_N = 0$), the relative error on the mth natural frequency is given by

$$\epsilon_{\text{nat freq}}^{O} = \left| \frac{\frac{m}{2} - \frac{N}{\pi} \sin \frac{m\pi}{2N}}{\frac{m}{2}} \right|, \quad m = 0, \cdots, N \tag{25}$$

and

$$\epsilon_{\text{nat freq}}^{S} = \left| \frac{\frac{1 + 2m}{4} - \frac{N}{\pi} \sin \left(\frac{1 + 2m}{4N} \pi\right)}{\frac{m}{4}} \right|, \quad m = 0, \cdots, N - 1 \tag{26}$$

in the open- and the short-circuited case respectively.

The relative errors on the natural frequencies are independent of the line length, d, in the lossless case, and they are shown in parts (a) and (b) of Fig. 3. The *useful frequency range* of a lumped element model is determined by the relative errors on the natural frequencies in the extreme loading conditions $Z_L = \infty$ and $Z_L = 0$. For example, a 16-T-cell model may be used for normalized frequencies, f_N, up to 1.75 if a relative error of 2% is tolerated on the natural frequencies of the line.

This error criterion is a frequency-domain criterion. It considers only the error on the pole location or on the natural frequency. The application of this criterion to the time domain is not obvious since transient signals may contain a wide spectrum of frequencies. All poles in the main part of the spectrum of the input signal must be accurately modeled. This error criterion gives some very useful information about the dynamic behavior of a lumped element model.

*3) Relative Error on the **ABCD** Matrix Coefficients:* The relative error on the *ABCD* coefficients can be interpreted more easily. The elements of the chain matrix are transfer functions which relate the voltages and the currents at both ends of the transmission line. If the elements of the ***ABCD*** matrix are approximated within a sufficient degree of accuracy, then the Z and Y parameters of the line, and thus the signal propagation for different loading conditions, are modeled accurately too. It is proved in [16] that a good approximation of the ***ABCD*** matrix also guarantees a good approximation of the S parameters (reflection and transmission behavior) for any real reference resistances.

In [15] and [16] a new criterion based on the *ABCD* coefficients was proposed which evaluates the accuracy of lumped element models independent of driving and load-

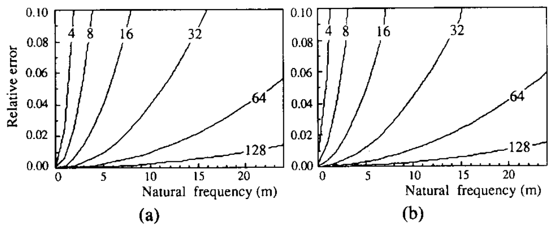


Fig. 3. Relative error on the natural frequencies for different numbers of cells: (a) open and (b) short-circuited end ($R_N = G_N = 0$).

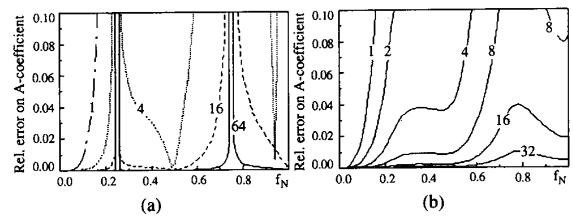


Fig. 4. Relative error on A coefficient for different numbers of cells: (a) $R_N = G_N = 0$ (lossless case); (b) $R_N = 1$ and $G_N = 0$.

ing impedances. This criterion is here extended to lines with nonnegligible conductance ($G_N > 0$), and the strong limitations in the (nearly) lossless case are toned down. In the Appendix, this criterion is also used to evaluate the accuracy of hybrid equivalent circuits.

We define the relative error on the A coefficient as

$$\epsilon_A = \left|\frac{A - A_{\text{T cells}}}{A}\right|. \tag{27}$$

This error is shown in parts (a) and (b) of Fig. 4 as a function of the normalized frequency, f_N, for different parameters R_N, G_N, and N. The relative errors on the other coefficients are defined in an analogous way. The influence of the zeros on the error functions is clear: if the zeros, given by (21) and (22), come closer to the imaginary axis in the plane of the normalized Laplace operator s_N (9), the relative errors ϵ_A, ϵ_B, and ϵ_C increase. In the lossless case ($R_N = G_N = 0$), the zeros lie on the imaginary f_N axis and the relative errors are infinite in these points. If the losses in the line increase, the zeros move into the left half-plane and their influence along the frequency axis decreases.

For different R_N and G_N values and for different numbers of cells, the normalized frequency, f_N, up to which a maximum tolerated value is not exceeded by one of the relative errors ϵ_A, ϵ_B and ϵ_C (worst-case frequency domain error criterion) can be determined. This maximal tolerated error is 5% in parts (a) and (b) of Fig. 5.

The simplest and least complex lumped element model which guarantees a desired accuracy can be selected with these results. The curves, drawn for different numbers of cells, guarantee an accuracy of at least 95% on voltage and current after one passage along the line for frequency components from dc up to the shown normalized frequency, f_N.

We see that a lossy transmission line can easily be modeled very accurately if the linelength is small compared with the wavelength of the signal or if $f_N << 1$. Even if the wavelength of the signal is relatively small compared with the line length ($0 << f_N < 1$), i.e., for high-frequency signals, a relatively small number of T cells (e.g., $N = 16$) is sufficient to model a real lossy transmission line.

The accuracy of the model decreases if multiple reflections occur because small propagation errors can accumulate if the signal travels several times up and down the

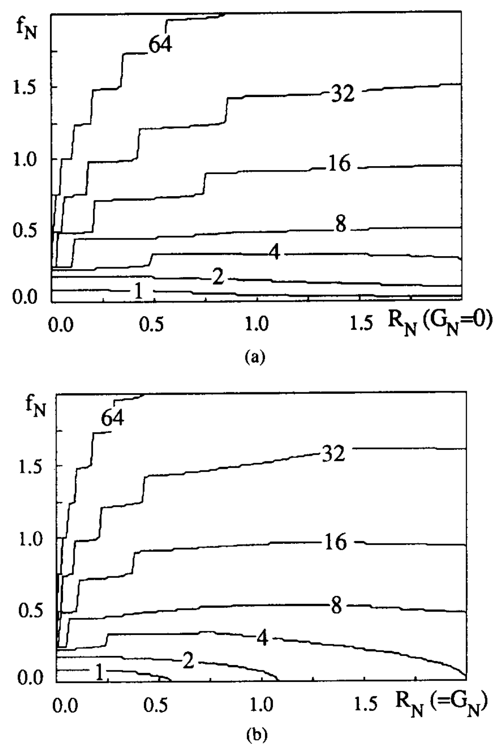


Fig. 5. Maximal normalized frequency, f_N, so that MAX(ϵ_A, ϵ_B, ϵ_C) ≤ 5%: (a) $G_N = 0$;(b) $R_N = G_N$.

line. However, the transmission lines used for high-speed interconnections are quite often matched or quasi-matched, and then the reflections are limited.

The steps in the curves of parts (a) and (b) of Fig. 5 are situated around the natural frequencies of the line. Increasing R_N and/or G_N, which pushes the zeros of the $ABCD$ coefficients into the left part of the s_N plane, decreases the relative error.

If a finite number of cells are used to model a lossless line, the relative error is always infinite for $f_N = 0.25$, because the approximated and the real zeros of the line do not match exactly for this particular normalized frequency. On the other hand one can see in Fig. 4(a), for example, that a 32-cell model can be used up to much higher frequencies, f_N, excluding one or more small frequency ranges. Normally only a small fraction of the frequency components of a signal are situated in these less accurately modeled ranges, so that the very severe limi-

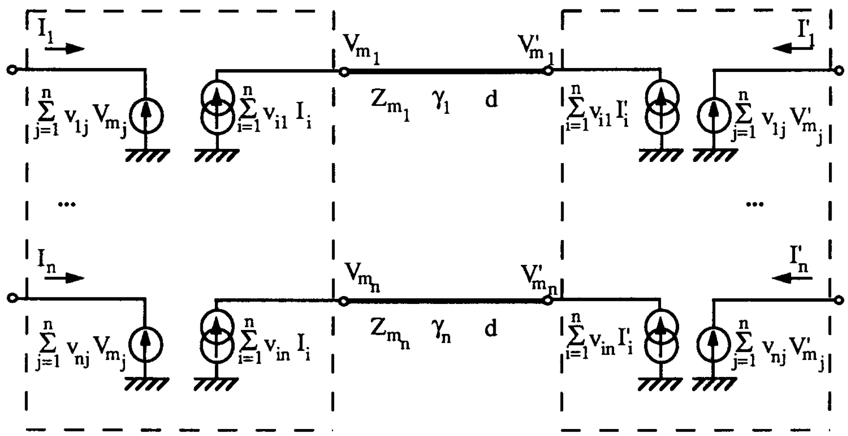


Fig. 6. Decoupled equivalent transmission line model.

tations of Fig. 5 must be toned down in the low-loss case. We conclude that in the low-loss case it is essential to observe the relative error on the natural frequencies (parts (a) and (b) of Fig. 3) to determine the useful frequency range of a lumped element model.

Many general-purpose circuit simulators can handle single lossless lines. A lossy transmission line can then be modeled as a combination of lumped elements and ideal lossless lines. In the Appendix the structure and the accuracy of these hybrid equivalent circuits is discussed. These models are especially well suited for slightly lossy lines!

III. Coupled Transmission Line Models

A. Decoupled Model

Consider a set of n coupled transmission lines. It is well known that the telegrapher's equations can be decoupled in n fundamental propagation modes. The decoupled mode analysis of multiple uniform coupled transmission lines is quite often used for transient analysis studies [11], [19], [20]. Each mode can be represented by a single transmission line.

The transmission line equations are given by

$$-\frac{d}{dz}\begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} 0 & Z \\ Y & 0 \end{bmatrix}\begin{bmatrix} V \\ I \end{bmatrix} \quad (28)$$

where $V(=[V_i]_{[n\times 1]})$ and $I(=[I_i]_{[n\times 1]})$ are the line voltage and current vectors. $Z(=[R_{ij}+j\omega L_{ij}]_{[n\times n]})$ and $Y(=[G_{ij}+j\omega C_{ij}]_{[n\times n]})$ represent the impedance and admittance matrices per unit length respectively. This can be rewritten in modal form as

$$-\frac{d}{dz}\begin{bmatrix} V_m \\ I_m \end{bmatrix} = \begin{bmatrix} 0 & \Lambda_m Z_m \\ \Lambda_m Y_m & 0 \end{bmatrix}\begin{bmatrix} V_m \\ I_m \end{bmatrix} \quad (29)$$

with

$$\begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} M_v & 0 \\ 0 & \{M_v^T\}^{-1} \end{bmatrix}\begin{bmatrix} V_m \\ I_m \end{bmatrix} \quad (30)$$

where $V_m(=[V_{m_i}]_{[n\times 1]})$ and $I_m(=[I_{m_i}]_{[n\times 1]})$ are the modal voltage and current vectors respectively.

The modal impedance matrix, $Z_m(=\text{diag}\,(Z_{m_i})_{[n\times n]})$, the modal propagation factor matrix, $\Lambda_m(=\text{diag}\,(\gamma_i)_{[n\times n]})$, and the voltage eigenvector matrix, M_v $(=[v_{ij}]_{[n\times n]})$ are related by

$$Z_m = (Y_m)^{-1} = \Lambda_m^{-1} M_v Z \{M_v^T\}^{-1} \quad (31)$$

and

$$ZYM_v = M_v \Lambda_m^2. \quad (32)$$

The equivalent circuit of the coupled lines is shown in Fig. 6. The model consists of two decoupling circuits and n single transmission lines. The modal decomposition blocks consist of real or complex linear dependent current and voltage sources. In many practical situations, for example n identical coupled lines, the elements of the voltage eigenvector matrix are real. Otherwise, the frequency-dependent voltage eigenvector matrix, M_v, might be approximated by a real constant matrix [21], [22]. Then the decoupling blocks can be exactly modeled in a general-purpose circuit simulator such as SPICE. The decoupled transmission lines propagate the n fundamental modes, and each line can be modeled with a desired accuracy by elementary *RGLC* cells (Fig. 1). The methods described above are used to determine the number of cells needed.

Our approach is an extension of the lossless models described in [19] and [20] and differs from [11] in that we used standard T cells to model the single lossy lines instead of a cascade of lumped elements and ideal lossless transmission lines. (The accuracy of such symmetric hybrid cells is discussed in the Appendix.)

The advantages of the decoupled model are that the decoupling is exact and that the same criteria and the same normalized parameters can be used for the propagating modes as the ones derived for the single transmission line.

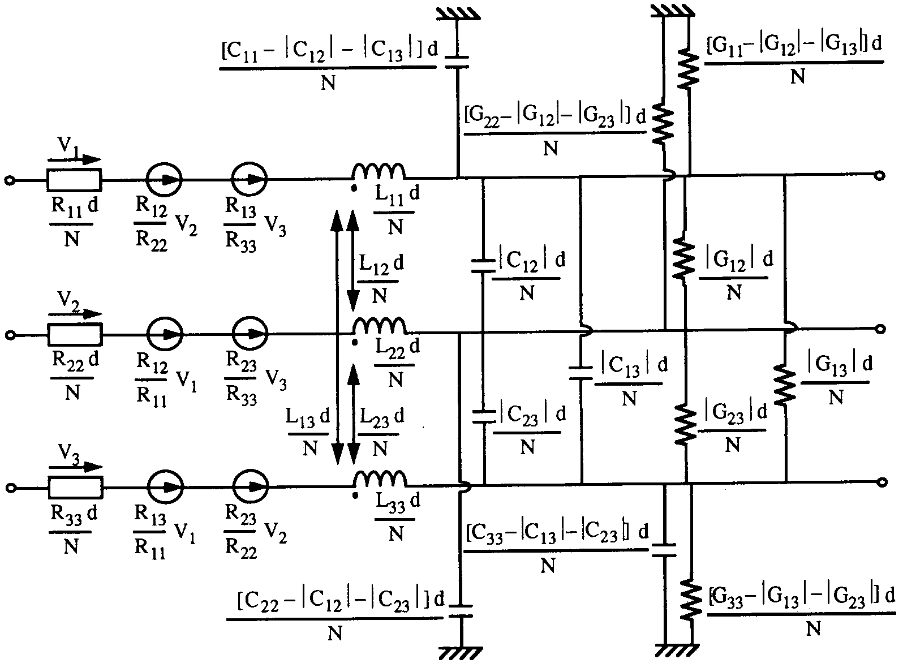


Fig. 7. Elementary coupled Γ cell.

B. Coupled Model

The coupled lumped element model can be derived from the telegrapher's equations [35] in a way analogous to that for the single line model. The partial differential equations have to be discretized (finite difference approximation) with respect to the space coordinate z. In Fig. 7 an elementary *RGLC* Γ cell for three coupled lines is shown. The elementary cell can have different forms (Γ, Π, T). The choice of the cell type is unimportant if a large (e.g., > 3) number of cells are used.

The coupling between the lines is modeled by inductive, capacitive, resistive, and conductive lumped elements. The resistive coupling is caused by the finite conductivity of the ground conductor and by the induced Foucault currents in the conductors. This resistive coupling is represented by voltage-dependent sources.

The criteria based on the *ABCD* matrix, derived above for a single line, can be used for the different propagating modes. A worst-case study of the results given an idea of the number of cells needed. The modeled cross talk will be somewhat lower owing to the discrete nature of the model. In the model, the energy interchange takes place in a finite number of points along the line instead of continuously. The discretization error can be limited by increasing the number of cells, which increases the complexity of the equivalent circuit.

IV. Numerical Results

To illustrate the use of the criteria derived above, we consider a lossy transmission line ($C = 50$ pF/m, $L = 500$ nH/m, $R = 1$ kΩ/m, $G = 0.1/\Omega$m, $d = 0.05$ m) driven by a pulse source and terminated in a 50 Ω resistor (Fig. 8). The source generates an alternating on–off bit pattern of 300 Mbit/s. The linear transition from 0 V to 1 V takes one tenth of a bit period. The rise time of the signal $t_{r_{10\%\text{-}90\%}}$ is equal to 0.26667 ns. If one uses the simple relationship $Bt_{r_{10\%\text{-}90\%}} = 0.35$, one gets a 3 dB bandwidth, B, of 1.3125 GHz. A more accurate method, based on a 99% coverage of the power spectrum [9], gives $B = 1.05$ GHz.

The normalized parameters of the transmission line are given by:

$$f_{N_{\text{MAX}}} = f_{\text{MAX}}\, d\sqrt{LC} = 0.2625$$

$$R_N = \frac{Rd}{\sqrt{L/C}} = 0.5 \qquad G_N = Gd\sqrt{\frac{L}{C}} = 0.5. \quad (33)$$

Note that the transmission line is distortionless ($R_N = G_N$), so the propagated signals maintain their shape, but their amplitude decreases exponentially. Fig. 5(b) shows that a 4-T-cell model can be used to model this line up to a normalized frequency, f_N, of 0.34 without exceeding the severe 5% error bound.

In Fig. 9 the results of a time-domain calculation [4] and a 4-T-cell model are compared. The delay time is modeled well, but a small overshoot and a slight ripple appears. The reason is that the transmission line is badly matched, and the signal is reflected several times. So small errors associated with each transmission and reflection add up.

As a second example, we consider the coupled transmission line structure of Fig. 10. This example is also

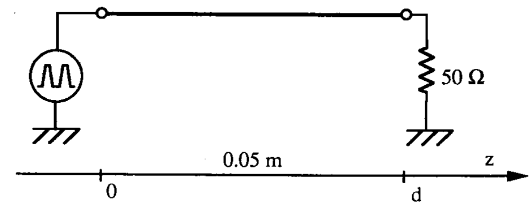


Fig. 8. Lossy distortionless transmission line structure.

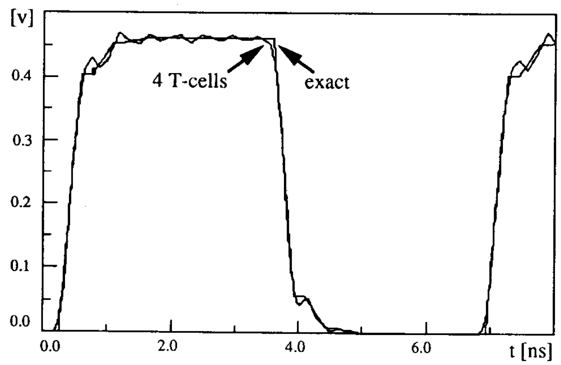


Fig. 9. Pulse response at load end (Fig. 8) calculated with a time-domain method [4] and with a lumped element model in SPICE.

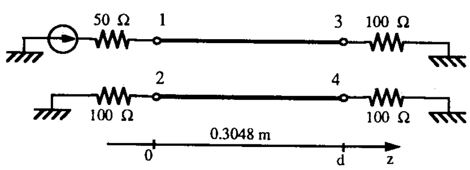


Fig. 10. Coupled transmission line structure.

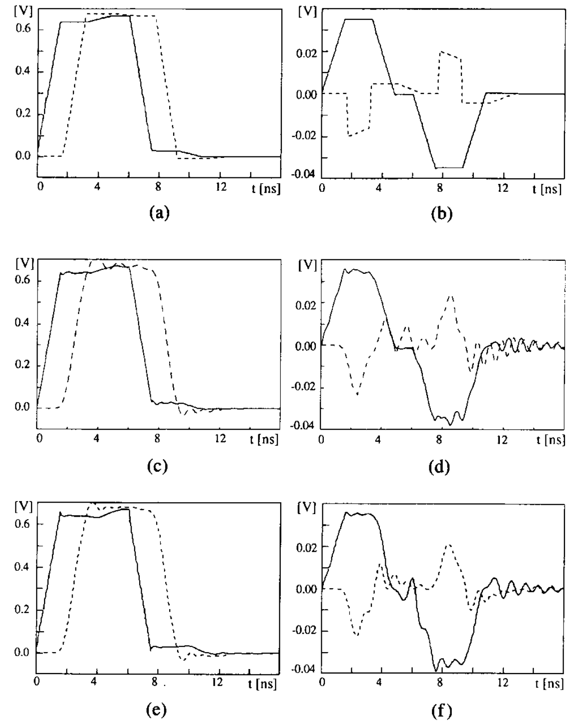


Fig. 11. Simulation of lossless lines (Fig. 10): ((a), (b)) time-domain method [4]; ((c), (d)) decoupled lumped element model; ((e), (f)) coupled lumped element model; ((a), (c), (e)) voltages on driven line; ((b), (d), (f)) voltages on senseline.

described in [4] and [23]. The internal impedance of the generator is 50 Ω. All other line ends are terminated in a 100 Ω impedance.

In the *lossless case*, the configuration is characterized by the line length $d = 0.3048$ m and the parameters

$$[L] = \begin{bmatrix} 494.6 & 63.3 \\ 63.3 & 494.6 \end{bmatrix} \text{nH/m}$$

$$[C] = \begin{bmatrix} 62.8 & -4.9 \\ -4.9 & 62.8 \end{bmatrix} \text{pF/m}. \tag{34}$$

The source generates an alternating on–off signal of 166.67Mbit/s. The rise and fall time is one quarter of the bitperiod. This corresponds to $B = 250$ MHz (99% coverage of the power spectrum) [9]. The maximal normalized frequencies of the even and odd modes are then, respectively, 0.43 and 0.41. Since $f_{N_{\max}} > 0.25$, some frequency components will be modeled with less precision. Based on Fig. 5, eight cells of the coupled model and eight cells for both lines of the decoupled model are chosen for the SPICE simulation. The simulation results of the decoupled lumped element model ((c) and (d)) and the coupled lumped element model ((e) and (f)) are compared in Fig. 11 with a simulation in the time domain ((a) and (b)) [4]. The figures on the left side ((a), (c), and (e)) represent the voltages at the ends of the driven line, and the other figures ((b), (d), and (f)) represent the forward and backward cross talk on the sense line. The full (dashed) lines show the voltage on the left (right) side of the network. Both lumped element models simulate the voltages on the active line quite accurately. Only a small overshoot appears.

The decoupled model models the two propagating modes separately. Adding and subtracting the modal signals gives respectively the voltages on the active and on the passive line. In both cases, however, the maximal possible *absolute* errors must be added. So the maximal possible *relative* errors on the voltages of the active line and on the modal voltages are the same, but the maximal possible *relative* error on the voltages of the passive line may be much higher! This problem occurs in all methods which use decoupling.

In the coupled model, the energy interchange takes place at a finite number of points along the line rather than continuously. This causes discretization ripples on the voltages of the sense line.

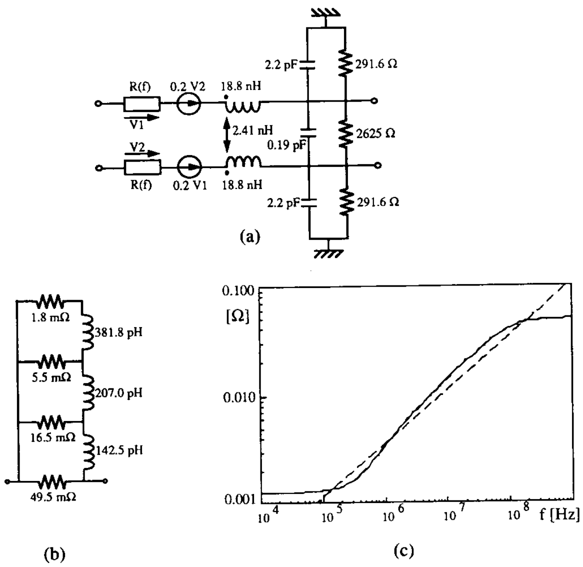


Fig. 12. (a) Elementary cell of coupled lumped model. (b) Equivalent circuit of $R(f)$. (c) Frequency behavior of this equivalent network compared with ideal $\sqrt{f}$ behavior.

Now, suppose that there are *frequency-dependent losses:*

$$[R] = \begin{bmatrix} 0.1 & 0.02 \\ 0.02 & 0.1 \end{bmatrix} \sqrt{f}\,\mathrm{m}\Omega/\mathrm{m}$$

$$[G] = \begin{bmatrix} 0.1 & -0.01 \\ -0.01 & 0.1 \end{bmatrix} \mathrm{S/m}. \qquad (35)$$

Owing to the skin-effect losses, the line resistance varies with the square root of the frequency. The frequency-dependent resistance is modeled by a chain network of resistors and inductors [6] in the desired frequency region (Fig. 12). The normalized parameters of the even and the odd mode are respectively $f_N = 0.43$, $G_N = 2.69$, $R_N = 3.73 \cdot 10^{-4}(f/\mathrm{MHz})^{0.5}$ and $f_N = 0.41$, $G_N = 2.68$, and $R_N = 3.06 \cdot 10^{-4}\ (f/\mathrm{MHz})^{0.5}$.

Based on a worst-case study of the *ABCD* error criterion, eight cells of the coupled model and eight cells for both lines of the decoupled model are chosen for the SPICE-simulation. This guarantees a maximal 5% error after one propagation along the line.

The scattering parameters (reference impedance = 50 Ω) of the coupled lumped element model (full lines) and of the ideal lossy dispersive coupled lines (dashed lines) are compared in Fig. 13 over the frequency range of interest (0–250 MHz). Note that a good approximation of the *ABCD* parameters guarantees a good approximation of the *S* parameters.

In Fig. 14 the simulation results of the decoupled lumped element model ((c) and (d)) and the coupled lumped element model ((e) and (f)) are compared with the results of a frequency-domain analysis ((a) and (b)) [22]. The results of the lumped element model are in good agreement with the frequency-domain method. Again, the signals on the sense line are somewhat less accurately simulated.

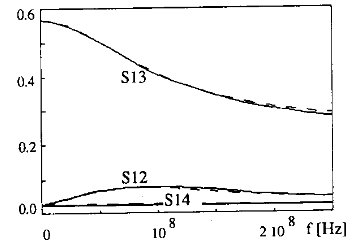


Fig. 13. Scattering parameters of coupled lumped element model (——) and of lossy dispersive coupled lines (—) over the modeled frequency range.

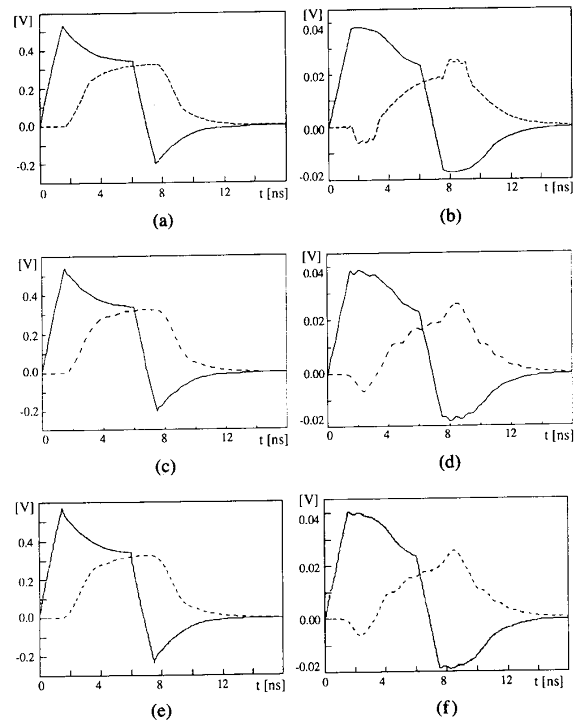


Fig. 14. Simulation of lossy lines (Fig. 10): ((a), (b)) frequency-domain method [23]; ((c), (d)) decoupled lumped element model; ((e), (f)) coupled lumped element model; ((a), (c), (e)) voltages on driven line; ((b), (d), (f)) voltages on sense line.

Since the poles and the zeros of the relevant transfer functions move into the left half-plane as the losses of the line increase, the lossy lines are modeled more accurately than the lossless lines.

V. Conclusion

Based on a dimensional analysis, three independent normalized transmission line parameters have been defined, and used throughout this paper. This makes it possible to use the results in a wide range of applications, varying from relatively short, high-speed electronic interconnections to long-distance power lines.

Useful criteria are developed for selecting the number of T cells or hybrid cells if a certain accuracy is desired for the characteristic impedance, the propagation factor, the natural frequencies of the line, and the signal propagation along the line. An often-used rule of thumb to select the number of cells needed [10], [13] is derived and evaluated. The error criterion based on the ***ABCD*** matrix is further developed and is shown to be very useful for CAD applications and transient studies. However, if losses are low, the error on the pole location may be considered as well.

A new decoupled lumped element model for coupled lossy lines is presented, which uses the same dimensionless parameters and the same criteria derived for a single line.

The major advantage of the lumped element approach is its simplicity in implementation and simulation. It gives reasonably good results quickly and without large programming efforts.

Appendix
Hybrid Cells

General-purpose circuit simulators, such as SPICE or ASTAP, can handle single lossless transmission lines. Normally, the well-known method of characteristics [24], [25] is used to model such lines. This simple line model can be used to build an elementary cell of a lossy transmission line (Fig. 15). The basic idea is to model a lossy line of length d as a cascade of N two-ports, each consisting of an ideal lossless line of length d/N and two resistive networks. The lossless line and the resistors model respectively the time delay and the losses. This hybrid cell, which consists of lumped and distributed elements, is an extension of [6] and [12].

The ***ABCD*** matrix of a hybrid cell is given by

$$\begin{bmatrix} \left(1+\frac{RG}{4N^2}\right) & \frac{R}{2N} \\ \frac{G}{2N} & 1 \end{bmatrix} \begin{bmatrix} \cos\frac{\omega d}{N} & j\sqrt{\frac{L}{C}}\sin\frac{\omega d}{N} \\ j\sqrt{\frac{C}{L}}\sin\frac{\omega d}{N} & \cos\frac{\omega d}{N} \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{R}{2N} \\ \frac{G}{2N} & \left(1+\frac{RG}{4N^2}\right) \end{bmatrix} \quad \text{(A.1)}$$

Note that $A_{\text{hybrid cell}} = D_{\text{hybrid cell}}$ and $\det(\boldsymbol{ABCD}) = 1$ owing to the symmetry and the reciprocity of the cell.

The accuracy of the hybrid equivalent circuits depends on the normalized line parameters (f_N, R_N, G_N) and on the number of cells (N). Error criteria equivalent to the ones derived for T cells can be used. For example, the relative error on the characteristic impedance is shown in parts (a) and (b) of Fig. 16 as a function of the normalized frequency, f_N, for different numbers of cells and for different parameters R_N and G_N. The *ABCD error criterion* is most suited for practical use. The maximal normalized frequency, f_N, up to which a tolerated error of 5% is not exceeded by one of the relative errors ϵ_A, ϵ_B, and ϵ_C is shown in parts (a) and (b) of Fig. 17 for different R_N, G_N, and N values. The simplest and least complex lumped element model which guarantees a desired accuracy can be selected with these figures. In parts (a) and (b) of Fig. 18, the maximal relative error in the *ABCD* coefficients in the frequency range $0 \le f_N \le 2$ is shown for different values of R_N, G_N, and N.

The hybrid cells are well suited for modeling lines with low losses. Of course, the lossless line is always modeled exactly by only one hybrid cell.

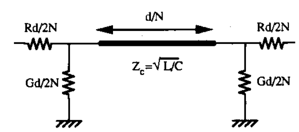


Fig. 15. Symmetric elementary hybrid cell.

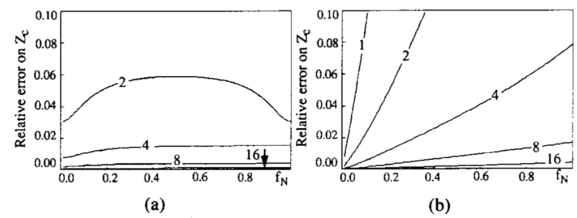
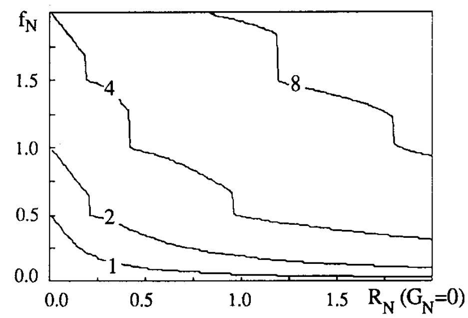
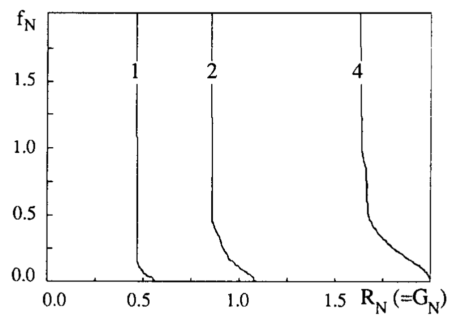


Fig. 16. Relative error on the characteristic impedance for different numbers of cells: (a) $R_N = G_N = 1$; (b) $R_N = 1$ and $G_N = 0$.



(a)



(b)

Fig. 17. Maximal normalized frequency, f_N, so that MAX(ϵ_A, ϵ_B, ϵ_C) $\le$ 5%: (a) $G_N = 0$; (b) $R_N = G_N$.

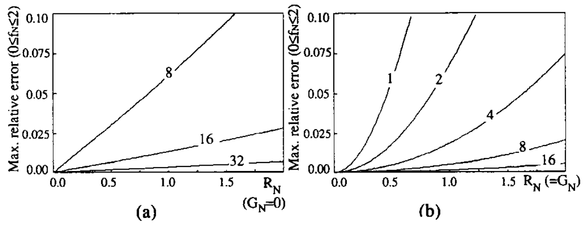


Fig. 18. Maximal relative error on *ABCD* coefficients in frequency range $0 \leq f_N \leq 2$ for different numbers of cells: (a) $G_N = 0$; (b) $R_N = G_N$.

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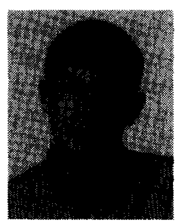

Tom Dhaene was born in Deinze, Belgium, on June 25,1966. He received the degree in electrical engineering from the University of Ghent, Belgium, in 1989. He is currently working toward the Ph.D. degree in electrical engineering at the Laboratory of Electromagnetism and Acoustics (LEA) of the same university. His research focuses on all aspects of circuit modeling and circuit simulation of high-frequency and high-speed interconnections.

Daniël De Zutter was born in Eeklo, Belgium, on November 8, 1953. He received a degree in electrical engineering from the University of Ghent in July 1976. From September 1976 to September 1984 he was a research and teaching assistant in the Laboratory of Electromagnetism and Acoustics (LEA) at the same university. In October 1981 he obtained the Ph.D. degree there and in the spring of 1984 he completed a thesis leading to a degree equivalent to the French Aggrégation or the German Habilitation.

He is now a professor at Ghent University and Research Director at the National Science Foundation of Belgium. Most of his earlier scientific work dealt with the electrodynamics of moving media, with emphasis on the Doppler effect and Lorentz forces. His research now focuses on circuit electromagnetic modeling of high-speed and high-frequency interconnections.

In 1990 Dr. De Zutter was elected as a member of the Electromagnetics Society.