CAD-Oriented General Circuit Description of Uniform Coupled Lossy Dispersive Waveguide Structures

Tom Dhaene and Daniel De Zutter

Abstract—**A general full-wave circuit description of uniform coupled lossy dispersive waveguide structures is presented. Starting from Maxwell's equations, the generalized coupled telegrapher's equations are directly found and the frequency dependent line parameter matrices $R(\omega)$, $G(\omega)$, $L(\omega)$ and $C(\omega)$ are defined in an unambiguous way. A new symmetric high-frequency characteristic impedance matrix $Z_c(\omega)$ is introduced which is much more suited for circuit simulation purposes than the traditional line-mode impedances. The reciprocity relation is explicitly taken into account. The complete power distribution over the different modes and different lines are calculated in a rigorous and concise way. The major advantage of this high-frequency model is its simple circuit-interpretation and its compatibility with the well-known quasi-static circuit models. The matrix formalism is used throughout this paper. This guarantees a compact, easily implementable and very general description which is well suited for CAD applications.**

I. Introduction

IN RECENT years, several authors have described loss-less and/or lossy uniform waveguide structures. Originally, the frequency dependence of the modal parameters was neglected and modeling was based on a quasi-TEM approximation [1]–[5]. However with increasing signal frequencies, the hybrid nature of the interconnections becomes more and more important [6]. A large number of publications deals with the calculation of the hybrid-mode characteristics (p.e., [7]–[8]). A frequency dependent circuit model is required if the dispersive nature of such an interconnection structure has to be taken into account.

The uncoupled hybrid waveguide structure is extensively described in the literature, see e.g., [9]–[10]. Fewer authors described the more general coupled structures. For such structures Jansen [11] introduced a quite often used line-mode characteristic impedance based on an approximative partial power definition. The proposed power distribution is only exact in the quasi-static limit and reduces to the one proposed in [12]. Later on Wiemer and Jansen [13] proposed a modified reciprocity-related line-mode characteristic impedance definition for lossless multiconductor hybrid-mode transmission lines. The work of Tripathi and Lee [14] and Carin and Webb [15] is also restricted to lossless structures. Faché and De Zutter [16] introduced a correct partial power definition for a lossless two-conductor structure. However, the extension to lossy and to multiconductor waveguide structures is rather sketchy and incomplete.

In this study the matrix formalism is used. This guarantees a very compact, easy implementable and general description. Some results of previous studies can be seen as a special case (quasi-static approximation, single line, lossless case) of this new universal approach. Based on Maxwell's equations, an accurate coupled transmission line model is proposed for the fundamental modes of a uniform coupled dispersive lossy waveguide structure. This general high-frequency circuit model together with other linear and/or nonlinear device models can be used for transient simulation [17] and for CAD applications.

First, the relations between the different propagating modes are examined and the transformation between modal parameters and circuit parameters is defined in an unambiguous way based on the power-current (PI-) formulation [18] for typical multilayer structures such as striplines and microstrips or on the power-voltage (PV-) formulation for coplanar structures. Then, the general frequency dependent telegrapher's equations proceed directly from Maxwell's equations. No quasi-static assumptions or extrapolations are required. The dispersive waveguide structure is completely characterized by the frequency dependent symmetric line parameter matrices $\boldsymbol{R}(\omega)$, $\boldsymbol{G}(\omega)$, $\boldsymbol{L}(\omega)$ and $\boldsymbol{C}(\omega)$ or by the characteristic impedance matrix $\boldsymbol{Z}_c(\omega)$ and the complex propagating factor matrix $\Lambda_v(\omega)$. The frequency dependent characteristic impedance matrix $\boldsymbol{Z}_c(\omega)$ is proven to be symmetric due to reciprocity. This high-frequency characteristic impedance definition is much more suited for circuit simulation purposes than the traditional line-mode impedances [11]–[16] and it corresponds with the well-known static circuit characteristic impedance matrix [1] in the low frequency limit. Furthermore, the electromagnetic (normalized) fields associated with each mode and each conductor are defined. The power distribution over the different modes and different lines is calculated in a rigorous and concise way.

Finally, an example illustrates this general circuit-oriented modeling approach.

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II. Modal Representation of the Fields in a General Waveguide Structure

Consider a general coupled lossy waveguide structure (see Fig. 1) consisting of $N + 1$ conductors in a lossless or lossy inhomogeneous dielectric. Each conductor has an arbitrary cross-section and is uniform along its length (longitudinal z direction). The $(N + 1)^{\text{th}}$ line is chosen as the reference or ground conductor.

We focus our attention on the N fundamental modes of the coupled multiconductor structure. A single transmission line will correspond with each mode under consideration. It is also possible to extend our approach to higher-order modes, but we will not consider this problem here.

In the sequel, the common time dependence exp $(j\omega t)$ will be omited and we will use the phasor notation.

The electromagnetic field associated with each hybrid mode can be divided in a longitudinal and a transversal component [9]. The global electromagnetic fields consist of the sum of the partial fields of the N fundamental modes:

$$\overline{E}(x, y, z) = \boldsymbol{V}_v(z)^T \overline{\boldsymbol{E}}_t^M(x, y) + R_0 \boldsymbol{I}_v(z)^T \overline{\boldsymbol{E}}_l^M(x, y)$$

$$\overline{H}(x, y, z) = \boldsymbol{I}_v(z)^T \overline{\boldsymbol{H}}_t^M(x, y) + \frac{1}{R_0} \boldsymbol{V}_v(z)^T \overline{\boldsymbol{H}}_l^M(x, y) \tag{1b}$$

where[1]

$$\begin{aligned}
\boldsymbol{V}_v(z) &= [V_{v_p}(z)]_{[N \times 1]} \\
\boldsymbol{I}_v(z) &= [I_{v_p}(z)]_{[N \times 1]} \\
\overline{\boldsymbol{E}}_t^M(x, y) &= [\overline{E}_{t_p}^M(x, y)]_{[N \times 1]} \\
\overline{\boldsymbol{E}}_l^M(x, y) &= [\overline{E}_{l_p}^M(x, y)]_{[N \times 1]} \\
\overline{\boldsymbol{H}}_t^M(x, y) &= [\overline{H}_{t_p}^M(x, y)]_{[N \times 1]} \\
\overline{\boldsymbol{H}}_l^M(x, y) &= [\overline{H}_{l_p}^M(x, y)]_{[N \times 1]}.
\end{aligned} \tag{2}$$

R_0 ($\cong$ 120 $\pi\Omega$) is the characteristic impedance of free-space. The transversal (subscript "t") and longitudinal (subscript "l") fields depend only on the transversal space-coordinates x and y. The dimension of these fields are m^{-1}. The parameters V_{v_p} and I_{v_p} ($p = 1, \cdots, N$) are the modal voltage and current of the pth mode and their dimension is volt and ampere respectively. They depend only upon the longitudinal space-coordinate z. Throughout this paper the subindex "v" indicates field quantities while the subindex "c" is used for circuit quantities.

Starting from Maxwell's equations and (1), it is shown in Appendix A that the modal voltages and currents are

[1]The bold faced symbols represent matrices, and the superscript accent "$^-$" is used to denote vectors.

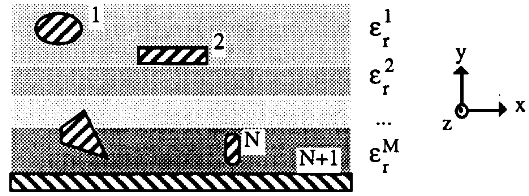

Fig. 1. General uniform waveguide structure with $N + 1$ conductors.

related by the following transmission line equations:

$$-\frac{d}{dz} \boldsymbol{V}_v(z) = \boldsymbol{\Gamma} \boldsymbol{Z}_v \boldsymbol{I}_v(z) \tag{3a}$$

$$-\frac{d}{dz} \boldsymbol{I}_v(z) = \boldsymbol{\Gamma} \boldsymbol{Z}_v^{-1} \boldsymbol{V}_v(z) \tag{3b}$$

where $\boldsymbol{Z}_v$ (= diag $[Z_{v_p}]_{[N \times N]}$) contains arbitrary complex modal impedances and $\boldsymbol{\Gamma}$ (= diag $[\gamma_p]_{[N \times N]}$) contains the complex modal propagation factors. The eigenvalue matrix-equation for these modal propagation factors is given in Appendix A.

The modal voltage and current vectors are the sum of wave components propagating in the negative and positive z-direction:

$$\boldsymbol{V}_v(z) = \boldsymbol{e}^{-\Gamma z} \boldsymbol{V}_v^+(0) + \boldsymbol{e}^{+\Gamma z} \boldsymbol{V}_v^-(0) = \boldsymbol{V}_v^+(z) + \boldsymbol{V}_v^-(z) \tag{4a}$$

$$\begin{aligned}
\boldsymbol{I}_v(z) &= \boldsymbol{e}^{-\Gamma z} \boldsymbol{I}_v^+(0) - \boldsymbol{e}^{+\Gamma z} \boldsymbol{I}_v^-(0) = \boldsymbol{I}_v^+(z) - \boldsymbol{I}_v^-(z) \\
&= \boldsymbol{Z}_v^{-1} [\boldsymbol{e}^{-\Gamma z} \boldsymbol{V}_v^+(0) - \boldsymbol{e}^{+\Gamma z} \boldsymbol{V}_v^-(0)]
\end{aligned} \tag{4b}$$

where the unknown constant vectors (at $z = 0$) are determined by the boundary conditions, i.e., by driving and receiving circuitry.

III. Transformation from a Modal Description to a Circuit Description

In order to simulate a hybrid structure with a circuit simulator [17] we have to transform the modal description into a circuit model consisting of coupled lossy dispersive transmission lines. For TEM structures, the conductor voltages and currents can be calculated in an unambiguous way by line-integrals of the electric and magnetic fields. For non-TEM structures however, there is no such unique definition of conductor voltage and current. Only in the quasi-static limit (quasi-TEM), both circuit parameters, voltage and current, have a unique and clear circuit interpretation. We use the well-accepted *PI-formulation* to model the structure under study. The circuit current $I_{c_i}(z)$ ($i = 1, \cdots, N$) is chosen to be identical to the total longitudinal current flowing along conductor i. This choice is the same as in the quasi-static case. Furthermore, both the circuit model representation and the real waveguide structure should have the same complex propagation modal factors γ_p and should propagate the same average complex power.

The discussion proceeds in three steps: first the circuit current is defined followed in a second step by the circuit voltage. The arbitrariness introduced in the first two steps is removed by the introduction of the conservation of power principle in a third step. As discussed in [18]–[19], the PI-model has the most TEM-like character for microstrips, striplines and related structures. Note however that the present study is not restricted to the PI-model! The discussion given in the sequel proceeds in a completely analogous way for the PV-formulation as explained in Appendix B.

A. Circuit Current

As stated above, the circuit current $I_{c_i}(z)$ ($i = 1, \cdots, N$) is defined as the total longitudinal current flowing along conductor i:

$$I_{c_i}(z) = \oint_i \overline{H}(x, y, z) \cdot d\overline{l} = \oint_i \boldsymbol{I}_v(z)^T \overline{\boldsymbol{H}}_t^M(x, y) \cdot d\overline{l} \tag{5}$$

with $i = 1, \cdots, N$. The current is a clear physical quantity that consists of contributions of the N propagating modes. The integration extends over the boundaries of the cross-section of conductor i (see Fig. 2). In the quasi-static limit the current is uniquely defined and the integration-path can be chosen arbitrary around the conductor, because the displacement current is zero.

Equation (5) can be written in a compact matrix-form as

$$\boldsymbol{I}_c(z) = \boldsymbol{M}_I \boldsymbol{I}_v(z) \tag{6a}$$

where $\boldsymbol{I}_c$ $(= [I_{c_i}(z)]_{[N\times 1]})$ is the circuit current vector and

$$\boldsymbol{M}_I = [I_{ip}]_{[N\times N]} \quad I_{ip} = \oint_i \overline{H}_{t_p}^M \cdot d\overline{l}; \tag{6b}$$

$\boldsymbol{M}_I$ is the frequency dependent transformation matrix between modal currents and circuit currents.

Remark that $\overline{H}_{t_p}^M$ can be multiplied with an arbitrary complex factor δ_p if I_{v_p} is divided by the same factor. Consequently, the modal-circuit transformation matrix $\boldsymbol{M}_I$ is NOT uniquely defined. On the other hand, the ratios I_{ip}/I_{jp} ($i, j, p = 1, \cdots, N$) are determined in an unambiguous way.

B. Circuit Voltage

The circuit voltage vector $\boldsymbol{V}_c$ $(= [V_{c_i}(z)]_{[N\times 1]})$ is still unknown. Now we will look for a practical representation suited for further matrix calculations. Later on, we will use the power-current (PI-) formulation to define the unknown parameters.

The electromagnetic fields can always be expressed as a linear combination of the propagating modal fields, hence, we can represent the circuit voltage vector $\boldsymbol{V}_c$ as a superposition of the modal voltages V_{v_p} ($p = 1, \cdots, N$):

$$\boldsymbol{V}_c(z) = \boldsymbol{M}_v \boldsymbol{V}_v(z) \tag{7}$$

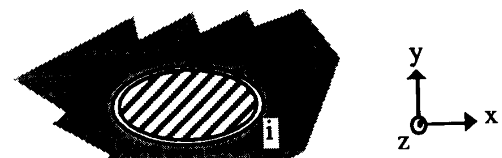

Fig. 2. Integration path around conductor i.

where $\boldsymbol{M}_v$ $(= [v_{ip}]_{[N\times N]})$ is the frequency dependent transformation matrix between modal voltages and circuit voltages.

With the help of (4) and (7), the voltage components propagating in the positive z-direction can be written as

$$\begin{aligned}\boldsymbol{V}_c^+(z) &= \boldsymbol{M}_v \boldsymbol{V}_v^+(z) = \boldsymbol{M}_v e^{-\Gamma z} \boldsymbol{V}_v^+(0)\\ &= \boldsymbol{M}_v \boldsymbol{Z}_v \boldsymbol{I}_v^+(z) = \boldsymbol{M}_v' \boldsymbol{I}_v^+(z)\end{aligned} \tag{8}$$

where $\boldsymbol{M}_v'$ $(= [V_{ip}]_{[N\times N]} = [v_{ip} Z_{v_p}])$ is the frequency dependent transformation matrix between modal currents and circuit voltages. The ratios V_{ip}/I_{jp} ($i, j, p = 1, \cdots, N$) are uniquely determined. For the negative z-direction, $\boldsymbol{M}_v'$ must be replaced by $-\boldsymbol{M}_v'$. In the sequel we will restrict ourselves to results for the $+z$-direction. In that case $\boldsymbol{I}_c^+(z)$ (6) and $\boldsymbol{V}_c^+(z)$ (8) are formed by the sum of individual contributions of the form $I_{ip} I_{v_p}^+(0) e^{-\gamma_p z}$ and $V_{ip} I_{v_p}^+(0) e^{-\gamma_p z}$ respectively. They can be seen as the contribution of the p^{th} mode to the circuit current and the circuit voltage propagating along the i^{th} conductor in the positive (longitudinal) z-direction. Only the first circuit factor has a real physical current meaning. The second one is defined via power and current (PI).

C. Power Conservation

We will now assign a value to the arbitrary matrices $\boldsymbol{M}_v$ and $\boldsymbol{M}_v'$ by looking at the propagated power. The circuit model and the coupled waveguide structure under study should propagate the same complex power. The *average complex power propagated by the coupled hybrid waveguide structure* can be found by integrating Poynting's vector over the cross-section S of the structure:

$$\begin{aligned}P_v^{\text{tot}} &= \tfrac{1}{2} \boldsymbol{V}_v(z)^T \iint_S [\overline{\boldsymbol{E}}_t^M(x, y) \times \overline{\boldsymbol{H}}_t^M(x, y)^{*T}] \cdot d\overline{S}\, \boldsymbol{I}_v(z)^*\\ &= \boldsymbol{V}_v(z)^T \boldsymbol{P}^{EH} \boldsymbol{I}_v(z)^*\end{aligned} \tag{9}$$

where $d\overline{S} = \overline{1}_z\, dS = \overline{1}_z\, dx\, dy$, and

$$\boldsymbol{P}^{EH} = [p_{pq}^{EH}]_{[N\times N]}$$

$$p_{pq}^{EH} = \tfrac{1}{2} \iint_S [\overline{E}_{t_p}^M(x, y) \times \overline{H}_{t_q}^M(x, y)^*] \cdot d\overline{S} \tag{10}$$

with $p, q = 1, \cdots, N$.

The average complex power propagating in the positive z-direction is

$$P_v^{\text{tot}+} = V_v^+(0)^T e^{-\Gamma z} P^{EH} e^{-\Gamma^* z} I_v^+(0)^* = I_v^+(z)^T P'^{EH} I_v^+(z)^* \tag{11}$$

where

$$P'^{EH} = [P_{pq}^{EH}]_{[N \times N]} = Z_v P^{EH}$$
$$P_{pq}^{EH} = p_{pq}^{EH} Z_{vp}. \tag{12}$$

On the other hand, the *average complex power propagated by the coupled transmission line model* is given by

$$P_c^{\text{tot}} = \tfrac{1}{2} V_c(z)^T I_c(z)^* = \tfrac{1}{2} V_v(z)^T M_v^T M_I^* I_v(z)^* \tag{13}$$

and the power propagating in the positive z-direction is

$$P_c^{\text{tot}+} = \tfrac{1}{2} V_c^+(z)^T I_c^+(z)^* = \tfrac{1}{2} I_v^+(z)^T M_v'^T M_I^* I_v^+(z)^* \tag{14}$$

From the right-hand side of (14) it follows the $P_c^{\text{tot}+}$ is constituted by elementary contributions of the form $I_{v_p}^+(0) e^{-\gamma_p z} P_{pq}^{EH} e^{-\gamma_q^* z} I_{v_p}^+(0)^*$ due to the electric field of mode p and the magnetic field of mode $q(p, q = 1, \cdots, N)$.

The average complex power propagated by the coupled hybrid waveguide structure and by the circuit model must be identical, hence $P^{\text{tot}} = P_c^{\text{tot}} = P_v^{\text{tot}}$. This requires that

$$\tfrac{1}{2} M_v^T M_I^* = P^{EH} \tag{15a}$$

or that

$$\tfrac{1}{2} M_v'^T M_I^* = P'^{EH} \tag{15b}$$

The equivalence of propagating power (15) defines the unknown matrices M_v and M_v'.

IV. Coupled Transmission Line Model

In this part our final transmission line model is introduced (Section IV-A). The generalized line parameter matrices $R(\omega)$, $G(\omega)$, $L(\omega)$ and $C(\omega)$ are defined. In Section IV-B it is shown how these matrices can be directly derived from the modal fields. Subsequent Sections (IV-D to IV-F) discuss the properties of our circuit model which can be seen as an extension of the more familiar quasi-TEM approximation. To this end so-called standard transformation matrices are introduced in Section IV-C. Finally the well-known quasi-static lossless "Chang"-circuit model [20] is extended to cover coupled lossy dispersive structures (Section IV-G).

A. Generalized Telegrapher's Equations

The generalized telegrapher's equations can be found by substitution of the circuit currents (6) and the circuit voltages (7) into the modal transmission line equations (3):

$$-\frac{d}{dz} V_c(z) = M_v \Gamma Z_v M_I^{-1} I_c(z) = Z_{\text{cir}} I_c(z) \tag{16a}$$

$$-\frac{d}{dz} I_c(z) = M_I \Gamma Z_v^{-1} M_v^{-1} V_c(z) = Y_{\text{cir}} V_c(z) \tag{16b}$$

Combining these two equations leads to the wave equations for the circuit currents and voltages:

$$\frac{d^2}{dz^2} V_c(z) = M_v \Gamma^2 M_v^{-1} V_c(z) = \Lambda_v^2 V_c(z) \tag{17a}$$

$$\frac{d^2}{dz^2} I_c(z) = M_I \Gamma^2 M_I^{-1} I_c(z) = \Lambda_I^2 I_c(z) = \Lambda_v^{2T} I_c(z) \tag{17b}$$

These (frequency dependent!) equations are very similar to the well-known static transmission line equations [1], [5]. Note however that no static extrapolations or assumptions were made.

The matrices $\Lambda_v(\omega) = M_v \Gamma M_v^{-1}$ and $\Lambda_I(\omega) = M_I \Gamma M_I^{-1}$ represent the complex voltage and current propagation matrix respectively. These matrices are uniquely defined. Note however that they are not symmetric.

B. Generalized Transmission Line Parameters

The matrices $Z_{\text{cir}}(\omega)$ and $Y_{\text{cir}}(\omega)$ are uniquely defined and they represent the circuit impedance and the admittance line matrices per unit length. Generally spoken, these matrices are frequency dependent, and we can split them in a real and an imaginary part:

$$Z_{\text{cir}}(\omega) = R(\omega) + j\omega L(\omega) = \Lambda_v(\omega) Z_c(\omega) \tag{18a}$$

$$Y_{\text{cir}}(\omega) = G(\omega) + j\omega C(\omega) = \Lambda_I(\omega) Y_c(\omega) \tag{18b}$$

where $R(\omega)$, $G(\omega)$, $L(\omega)$ and $C(\omega)$ are the generalized resistance, capacitance, conductance and inductance $[N \times N]$-matrices. The coupled transmission line model is completely characterized by $N(N + 1)$ complex frequency dependent line parameter.

The new circuit model is fully compatible with, and is an extension towards higher frequencies of the well-known TEM and quasi-TEM circuit models [1]. Some typical static concepts, such as capacitance and inductance, are generalized and introduced in the high-frequency model. Note that at each discrete frequency the model can be seen as a quasi-static model. The resemblance with the quasi-static model is the main reason why the proposed frequency dependent model is perfectly suited for CAD calculations.

The line parameter matrices $R(\omega)$, $G(\omega)$, $L(\omega)$ and $C(\omega)$, and also $Z_{\text{cir}}(\omega)$ and $Y_{\text{cir}}(\omega)$, are symmetric because the coupled waveguide structures are reciprocal (at least in the absence of anisotropic media). Taking relation (16) into account, the symmetry of Z_{cir} and Y_{cir} leads to the following important reciprocity relation:

$$M_I^T M_v' = \text{diag}\,(b_p^2) \tag{19}$$

The diagonal matrix diag $(b_p^2)_{[N \times N]}$ depends on power and current. Combining (15b) and (19) leads to

$$2P'^{EH} = \text{diag}\,(b_p^2) M_I^{-1} M_I^* \tag{20}$$

In the lossless case the transformation matrix M_I can always be chosen to be real. In that case (20) reduces to

$$2P'^{EH} = \text{diag}\,(b_p^2) \tag{21}$$

The power matrix P'^{EH} is diagonal in the lossless case, which means that the different propagating modes do not interchange any power: the modes are power-orthogonal. This can also be shown starting from Lorentz reciprocity relation. The diagonal elements of P'^{EH} give an indication of the average propagated power by the different modes.

It is very important to emphase that only in the lossless case the reciprocity relation (19) is included in the power relation (15)! This means that the reciprocity property is not included in the circuit model for lossy waveguide structures proposed in [16].

C. Standard Transformation Matrices

As mentioned earlier, the transformation matrices M_I and M'_v are not uniquely defined. If the partial modal field $\overline{H}_{tp}^M$ is multiplied with a arbitrary complex factor δ_p, the factors I_{ip} and V_{ip} are also multiplied with the same factor. Based on the reciprocity relation (19), we introduce two new standard transformation matrices:

$$M_I^{\text{standard}} = M_I\,\text{diag}\,(b_p^{-1}) \tag{22a}$$

$$M_v^{\text{standard}} = M'_v\,\text{diag}\,(b_p^{-1}) \tag{22b}$$

(19) then leads to the useful relation:

$$[M_I^{\text{standard}}]^T = [M_v^{\text{standard}}]^{-1}. \tag{23}$$

D. Characteristic Impedance Matrix

The frequency dependent characteristic impedance matrix $Z_c(\omega)$ is defined as

$$Z_c = \Lambda_v^{-1} Z_{\text{cir}} = M'_v M_I^{-1} = M_v^{\text{standard}} [M_I^{\text{standard}}]^{-1}. \tag{24}$$

This $[N \times N]$-matrix can be seen as the input impedance matrix of an infinitely long coupled transmission line structure. The characteristic impedance matrix relates the current waves to the voltage waves traveling in positive longitudinal direction:

$$V_c^+(z) = Z_c I_c^+(z) \tag{25}$$

Combining (15) and (24) leads to a familiar-looking power relation:

$$P'^{EH} = \tfrac{1}{2} M_I^T Z_c M_I^* = \tfrac{1}{2} M_v'^T M_I^* \tag{26}$$

In the lossless case M_I can always be chosen to be real. As M_I is real, (23) and (24) establish that the matrix Z_c is real, symmetric and has positive diagonal elements. Furthermore, having the form $A^T A$, with A a real $[N \times N]$-matrix, the characteristic impedance matrix is also positive definite [1].

The characteristic admittance matrix $Y_c(\omega)$ is defined in an analogous way:

$$Y_c = \Lambda_I^{-1} Y_{\text{cir}} = M_I M_v'^{-1} = Z_c^{-1} \tag{27}$$

This matrix is also symmetric. Both the characteristic impedance and admittance matrix do not depend upon the longitudinal space coordinate z and they are defined in an unambiguous way.

In the quasi-static limit, the new high-frequency characteristic impedance definition corresponds with the well-known (static) characteristic impedance matrix [1], [5]. It is much more suited for circuit simulation, and CAD applications than the traditionally used line-mode impedances [11]–[16]. The line-mode characteristic impedance Z_{ip} of a conductor i for an eigenmode p is defined as the ratio of the circuit voltage V_{ip} to the circuit current I_{ip} propagating in the z-direction along transmission line i, for mode p, thus:

$$Z_{ip} = \frac{V_{ip}}{I_{ip}} = \frac{v_{ip}}{I_{ip}} Z_{v_p}. \tag{28}$$

The relation between the characteristic impedance matrix $Z_c(\omega)$ and the line-mode characteristic impedance Z_{ip} is clearly expressed by (24):

$$[Z_{c_{ij}}]_{[N\times N]}\,[I_{jp}]_{[N\times N]} = [V_{ip}]_{[N\times N]} = [Z_{ip} \quad I_{ip}]_{[N\times N]}. \tag{29}$$

In the lossless case, the line-mode characteristic impedance Z_{ip} can be negative, as was shown theoretically along with some numerical examples in [15].

E. Wave Representation

The circuit voltage and current vectors are the sum of wave components propagating in negative and positive z-direction:

$$\begin{aligned} I_c(z) &= M_I(e^{-\Gamma z} I_v^+(0) - e^{+\Gamma z} I_v^-(0)) \\ &= e^{-\Lambda_I z} I_c^+(0) - e^{+\Lambda_I z} I_c^-(0) \end{aligned} \tag{30a}$$

and

$$\begin{aligned} V_c(z) &= M'_v(e^{-\Gamma z} I_v^+(0) + e^{+\Gamma z} I_v^-(0)) \\ &= Z_c(e^{-\Lambda_I z} I_c^+(0) + e^{+\Lambda_I z} I_c^-(0)) \end{aligned} \tag{30b}$$

where the unknown line vectors are determined by the boundary conditions.

F. Eigenvectors

Based on the generalized telegrapher's equations (16), it can be seen that the columns of the transformation matrices M_v and M_I respectively consist of the eigenvectors of the complex propagation matrix $\Lambda_v(\omega)$ and $\Lambda_I(\omega)$. The eigenvalue equations are found to be:

$$(Z_{\text{cir}} Y_{\text{cir}}) V_{\text{col}}^p = \gamma_p^2 V_{\text{col}}^p \tag{31}$$

and

$$I_{\text{col}}^{p^T} (Z_{\text{cir}} Y_{\text{cir}}) = \gamma_p^2 I_{\text{col}}^{p^T} \tag{32}$$

V_{col}^p and I_{col}^p are the right-hand and left-hand eigenvectors of Λ_v, associated with the same eigenvalue γ_p.

Both sets of eigenvectors V_{col}^p and I_{col}^p ($p = 1, \cdots, N$) span the complete N-dimensional space. Hence any

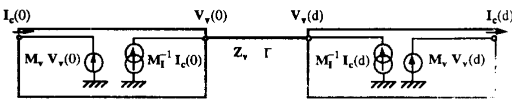

Fig. 3. Equivalent circuit model.

arbitrary voltage or current distribution can be represented as a weighted sum of these eigenvectors (see for example (6) and (7)).

The reciprocity relation (19) shows that the eigenvectors V^p_{col} and I^q_{col} are orthogonal, i.e. $V^{p^T}_{\text{col}} \cdot I^q_{\text{col}} = 0$ for $p \neq q$. This can also be proven by combining (31) and (32) in a proper way [1]. The standard eigencurrent and eigenvoltage vectors (22) are always ortho*normal*.

G. High-frequency Circuit Model for Coupled Lossy Dispersive Waveguides

In this part, we will generalize the quasi-static circuit representation for lossless transmission lines proposed by Chang [20], and construct a high-frequency circuit model for coupled lossy dispersive waveguides.

The transformation matrices M_I (6) and M_v (7) relate respectively the circuit to the modal currents (I_c and I_v) and the circuit to the modal voltages (V_c and V_v). We use these transformation matrices to decouple the transmission lines. In that way, the propagation of the N fundamental modes is represented by N single decoupled lines. The characteristic impedance of these lines depends upon the choice of the transformation matrices M_I and M_v. If we use the standard transformation matrices defined above, the characteristic impedance of the uncoupled lines is 1 Ω.

The equivalent (frequency domain) circuit representation of the general waveguide structure is shown in Fig. 3. Note that the multiplications in the frequency domain correspond with convolutions in the time domain! The general waveguide structure with N fundamental propagating modes, is completely characterized by $N(N + 1)$ complex frequency dependent parameters. These parameters can be the N^2 line-mode characteristic impedances Z_{ip} (28) and the N modal propagation factors γ_p (3) which follow directly from the full-wave analysis. The $N(N + 1)/2$ relevant parameters of the symmetric circuit impedance matrix Z_{cir} (18a) and the circuit admittance matrix Y_{cir} (18b) are more suited for CAD-applications.

A lossless hybrid waveguide structure consisting of two signal conductors and one reference conductor ($N = 2$) for example is completely characterized by 3 frequency dependent generalized inductance (L_{11}, L_{22}, $L_m = L_{21} = L_{12}$) and 3 generalized capacitance (C_{11}, C_{22}, $C_m = C_{21} = C_{12}$) parameters. Remark the analogy with the (frequency independent) quasi-TEM description.

V. Power Distribution Over the Different Lines

In this section a detailed study is presented of the contribution of each mode to the power propagated by each line. The problem is complicated by the fact that the modes are no longer power orthogonal as is the case for lossless structures.

In (1), the global electromagnetic fields were represented as a sum of partial modal fields. This led to the power distribution over the different modes (P^{EH}). Now we will examine the power distribution over the different lines. Using (6) and (7), the global electromagnetic fields can be expressed as a function of circuit-related parameters:

$$\overline{E}(x, y, z) = V_c(z)^T \overline{E}^c_t(x, y) + R_0 I_c(z)^T \overline{E}^c_l(x, y) \tag{33a}$$

$$\overline{H}(x, y, z) = I_c(z)^T \overline{H}^c_t(x, y) + \frac{1}{R_0} V_c(z)^T \overline{H}^c_l(x, y) \tag{33b}$$

where

$$\overline{E}^c_t(x, y) = M_v^{T^{-1}} \overline{E}^M_t(x, y) \quad \overline{E}^c_l(x, y) = M_I^{T^{-1}} \overline{E}^M_l(x, y)$$
$$\overline{H}^c_t(x, y) = M_I^{T^{-1}} \overline{H}^M_t(x, y) \quad \overline{H}^c_l(x, y) = M_v^{T^{-1}} \overline{H}^M_l(x, y). \tag{34}$$

With each conductor corresponds an unique transversal and longitudinal normalized electric and magnetic field. Equation (34) shows that these normalized fields can be seen as a weighted average of the different propagating modal fields. In what follows, we only consider the waves propagating in positive z-direction.

Based on (14) and (33), we can prove that the partial transversal electromagnetic fields $\overline{E}^c_{t_i}(x, y)$ and $\overline{H}^c_{t_j}(x, y)$, associated with the conductors i and j, are power-orthonormal, i.e.:

$$\iint_S [\overline{E}^c_t(x, y) \times \overline{H}^c_t(x, y)^{*^T}] \cdot d\overline{S} = \mathbf{1}^{\text{diag}} \tag{35}$$

The circuit-related electromagnetic fields (34) are always power-orthonormal. On the other hand, the mode-related electromagnetic fields (2) are only power-orthogonal in the lossless case.

The current flowing along conductor i (5) can be expressed as a function of the transversal partial magnetic field:

$$I_{c_i}(z) = \oint_i I_c(z)^T \overline{H}^c_t(x, y) \cdot d\overline{l} \tag{36}$$

where the integration extends over the boundary of the cross-section of conductor i (see Fig. 2). This leads to:

$$\oint_i \overline{H}^c_{t_j}(x, y) \cdot d\overline{l} = \delta_{ij} \tag{37}$$

where δ_{ij} is the Kronecker delta. Note that the total electric and magnetic fields consist of longitudinal and transversal components, thus $\overline{1}_z \times \overline{H}^c_j \neq 0$.

The normalized field $\overline{H}^c_{t_i}(x, y)$ is a weighted average of the propagating modal transversal magnetic fields (34).

This field is responsible for a current of 1 A along conductor i, and for a current of 0 A along the other lines in the circuit model (37). The current of 0 A should be interpreted in the average sense, this means that some longitudinal currents may flow along conductor j ($\neq i$), but the net longitudinal current (= weighted sum of different modal currents) is identically zero.

Taking into account the power-orthogonality relation (35), we see that $\overline{E}_{t_i}^c(x, y)$ is responsible for a voltage of 1 V on conductor i and a voltage of 0 V on the other conductors of the circuit model.

We can conclude that the circuit-related fields are normalized in the average sense defined above.

Now we will split the electromagnetic fields associated with a conductor in smaller parts, each part associated with a different mode. We only consider the transversal fields. The calculations of the longitudinal fields proceeds in an analogous way. We define:

$$\overline{\boldsymbol{E}}_t^{CM+}(x, y) = \overline{\boldsymbol{E}}_t^{c\text{-diag}}(x, y)\boldsymbol{M}_V' \tag{38a}$$

$$\overline{\boldsymbol{H}}_t^{CM+}(x, y) = \overline{\boldsymbol{H}}_t^{c\text{-diag}}(x, y)\boldsymbol{M}_I \tag{38b}$$

where

$$\overline{\boldsymbol{E}}_t^{c\text{-diag}}(x, y) = \operatorname{diag}\,[\overline{E}_{t_i}^c(x, y)]_{[N\times N]}$$

and

$$\overline{\boldsymbol{H}}_t^{c\text{-diag}}(x, y) = \operatorname{diag}\,[\overline{H}_{t_i}^c(x, y)]_{[N\times N]}.$$

$\overline{E}_{tip}^{CM+}(x, y)$ ($\overline{H}_{tip}^{CM+}(x, y)$) is the transversal electric (magnetic) field associated with the pth mode and the ith conductor. From (37) and (38) we learn that

$$\delta_{ij} I_{ip} = \oint_i \overline{H}_{tjp}^{CM+}(x, y) \cdot d\overline{l} \tag{39}$$

where I_{ip} is defined in (6).

Substituting (38) in (35) leads to a new power distribution relation:

$$\frac{1}{2}\iint_S [\overline{E}_{tip}^{CM+} \times \overline{H}_{tjq}^{CM+*}] \cdot d\overline{S} = \frac{1}{2}\,\delta_{ij} V_{ip} I_{jq}^* = \delta_{ij} P_{ipq}^{CVI+} \tag{40}$$

where $I_{v_p}^+(0)e^{-\gamma_p z} P_{ipq}^{CVI+} e^{-\gamma_q^* z} I_{v_q}^+(0)^*$ is the average complex power propagating in positive z-direction along conductor i and associated with the magnetic field of mode q and the electric field of mode p.

Based on the power orthonormality relation of the partial fields $\overline{E}_{t_i}^c(x, y)$ and $\overline{H}_{t_j}^c(x, y)$ (38) and from (34), the partial power factor P_{ipq}^{CVI+} (40) can be expressed as

$$\begin{aligned} P_{ipq}^{CVI+} &= \frac{1}{2}\iint_S [\overline{E}_{tp}^{M} \times \overline{H}_{tiq}^{CM+*}] \cdot d\overline{S} \\ &= \frac{1}{2}\iint_S [\overline{E}_{tip}^{CM+} \times \overline{H}_{tq}^{M*}] \cdot d\overline{S}. \end{aligned} \tag{41}$$

In the lossless case and if $p = q$, this formula is similar to the approximate partial power expression proposed by Jansen [11] for coupled hybrid waveguides. Jansen's formulation was inspired by the (correct) power distribution for TEM-waveguides [12] and is only exact in the quasi-static limit. The partial magnetic field associated with conductor i and mode p used by Jansen ensures that NO longitudinal current will flow along conductor j ($\neq i$). On the other hand, the partial magnetic field $\overline{H}_{tip}^{CM+*}$ (38) used in this paper only ensures that the WEIGHTED SUM of the modal longitudinal currents flowing along conductor j ($\neq i$) is equal to zero (39). Hence, some modal longitudinal current-movement is allowed.

The definition of Jansen can only be used in the quasi-TEM limit, when the longitudinal electromagnetic fields are negligibly small. Our definition is appropriate above the quasi-TEM limit.

Based on (10), we write

$$P_{pq}^{EH} = \sum_{i=1}^{N} P_{ipq}^{CVI+} = \frac{1}{2}\sum_{i=1}^{N} V_{ip} I_{iq}^* \tag{42}$$

where P_{pq}^{EH} is proportional to the average complex power propagating in positive z-direction and associated with the magnetic field of mode q and the electric field of mode p (with $p, q = 1, \cdots, N$).

The total average power propagating in the positive z-direction (14) is given by

$$\begin{aligned} P^{\text{tot}+} &= \sum_{q=1}^{N}\sum_{p=1}^{N}\sum_{i=1}^{N} I_{v_p}^+(z) P_{ipq}^{CVI+} I_{v_p}^+(z)^* \\ &= \sum_{q=1}^{N}\sum_{p=1}^{N} I_{v_p}^+(z) P_{pq}^{EH} I_{v_q}^+(z)^* \end{aligned} \tag{43}$$

In the lossless case, the power orthonormality of the eigenmodes ensures that $P_{pq}^{EH} = 0$ if $p \neq q$. The average complex power associated with the electromagnetic fields of the pth mode and propagating in positive z-direction along the ith conductor of the lossless transmission line structure is given by $|I_{v_p}^+(z)|^2 P_{ip}^{CM}$ with $P_{ip}^{CM} = P_{ipp}^{CVI+} = \frac{1}{2} V_{ip} I_{ip}^*$.

If the structure is lossless and if $N = 2$, it can be shown that (40) and (44) are equivalent to

$$\frac{P_{111}^{CVI+}}{P_{211}^{CVI+}} = \frac{P_{222}^{CVI+}}{P_{122}^{CVI+}} = \frac{P_{11}^{CM}}{P_{21}^{CM}} = \frac{P_{22}^{CM}}{P_{12}^{CM}} \tag{45}$$

and

$$\frac{P_{112}^{CVI+}}{P_{212}^{CVI+}} = \frac{P_{221}^{CVI+}}{P_{121}^{CVI+}} = -1. \tag{46}$$

Note that no static assumptions or extrapolations were made in the power-analysis derived above. The power distribution over the different modes and different lines is calculated in a rigorous and concise way.

VI. Numerical Example

In this section, we construct an equivalent high-frequency transmission line model for a coupled two-line system laying in an inhomogeneous lossy medium. The

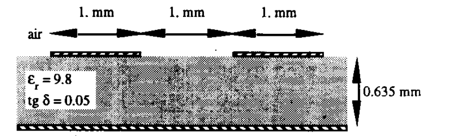

Fig. 4. Cross-section of a lossy coupled strip configuration.

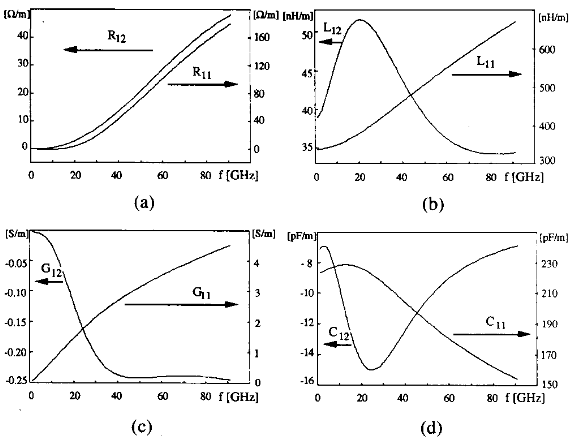

Fig. 5. Relevant elements of (a) resistance, (b) inductance, (c) conductance, and (d) capacitance matrix as a function of frequency.

cross-section of the strip configuration is shown in Fig. 4. The structure consists of a perfectly conducting ground plane, a lossy dielectric substrate (*tg* $\delta = 0.05$, $\epsilon_r = 9.8$), and a half-infinite air top-layer. The strips are infinitely thin and perfectly conducting. The width of both strips is 1 mm.

Two fundamental modes can propagate in this two-line system: an even mode and an odd mode. Using a rigorous full-wave integral equation technique [21], the modal propagation factors and the line-mode characteristic impedances are calculated in the frequency range 0–100 GHz. All losses are calculated in an exact way without making any approximations or perturbations. In Fig. 5, the elements of the symmetric resistance, inductance, conductance and capacitance matrices are shown as a function of frequency. Note the significant frequency dependence of all these parameters.

As expected, the resistive losses are relatively low, and the conductance factor G_{11} is rather high due to the conductance of the dielectric layer (*tg* $\delta = 0.05$). The electromagnetic fields concentrate more and more in the lossy dielectric layer as the frequency increases. Hence, L_{11} increases and C_{11} decreases with frequency. The frequency dependent elements of the characteristic impedance matrix $\boldsymbol{Z}_c(\omega)$ are shown in Fig. 6.

Finally, the equivalent transmission line model of the two-line structure under study is shown in Fig. 7. The propagation of the fundamental modes is represented by two single transmission lines. Both decoupled lines have a characteristic impedance of 1 Ω. This high-frequency circuit model together with other linear and/or nonlinear device models can be used for time-domain simulation [17] and for CAD applications.

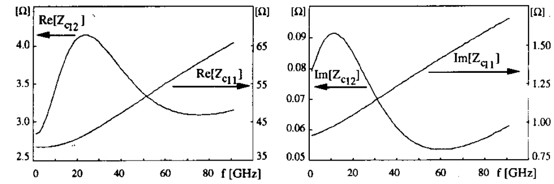

Fig. 6. Elements of characteristic impedance matrix as a function of frequency ($Z_{c11} = Z_{c22}$, $Z_{c12} = Z_{c21}$).

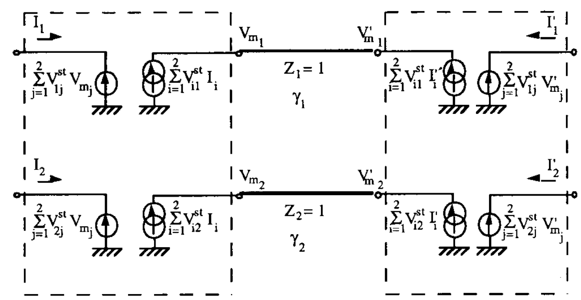

Fig. 7. Equivalent circuit representation of the coupled structure of Fig. 4.

VII. Conclusion

Starting from Maxwell's equations, an accurate coupled transmission line model has been proposed for the fundamental modes of a uniform coupled dispersive lossy waveguide structure. The frequency dependent circuit model is well suited for CAD applications and for implementation in circuit simulators. The matrix formalinm has been used throughout this paper, which guarantees a very clear, compact and general description.

Based on the well-accepted power-current (PI-) formulation for stripline or microstrip like structures or on the power-voltage (PV-) formulation for coplanar structures, the transformation between modal parameters and circuit parameters has been defined in an unambiguous way. The generalized telegrapher's equations have been derived directly from Maxwell's equations. This was also the case for the frequency dependent line parameter matrices $\boldsymbol{R}(\omega)$, $\boldsymbol{G}(\omega)$, $\boldsymbol{L}(\omega)$ and $\boldsymbol{C}(\omega)$, the complex propagating factor $\Lambda_v(\omega)$ and the symmetric characteristic impedance matrix $\boldsymbol{Z}_c(\omega)$. This new characteristic impedance definition is much more suited for circuit simulation purposes than the traditional line-mode impedances. The major advantage of this high-frequency model is its simple circuit-interpretation and its compatibility with the well-known quasi-static circuit models. Furthermore, the electromagnetic fields associated with each mode and each conductor have been defined. The fields associated with the conductors are always power-orthonormal. The power distribution over the different modes and different lines has been calculated in a rigorous and concise way.

Appendix A
Generalized Transmission Line Equations

In [9], relations are derived between the modal current and the modal voltage, and between the transversal and the longitudinal field components of a single transmission line. In this Appendix we derive analogous relations for the N fundamental modes of a coupled waveguide structure.

Our starting point is the total electromagnetic field as given by (1a) and (1b). We substitute these representations in Maxwell's equations, and separate the longitudinal and the transverse components. This leads to the modal transmission line equations (3a) and (3b), but also to the following vector-relations between the longitudinal and the transversal partial electromagnetic fields:

$$\nabla \times \overline{E}_t^M(x, y) + j\omega \frac{\mu(x, y)}{R_0} \overline{H}_l^M(x, y) = 0 \tag{A1a}$$

$$\Gamma Z_v \bar{1}_z \times \overline{E}_t^M(x, y) - R_0 \nabla \times \overline{E}_l^M(x, y) - j\omega\mu(x, y) \overline{H}_t^M(x, y) = 0 \tag{A1b}$$

$$\nabla \times \overline{H}_t^M(x, y) - j\omega\epsilon(x, y) R_0 \overline{E}_l^M(x, y) = 0 \tag{A2a}$$

$$\Gamma Z_v^{-1} \bar{1}_z \times \overline{H}_t^M(x, y) - \frac{\nabla \times \overline{H}_l^M(x, y)}{R_0} + j\omega\epsilon(x, y) \overline{E}_t^M(x, y) = 0 \tag{A2b}$$

$$\nabla \cdot \epsilon(x, y) \overline{E}_t^M(x, y) - R_0 \epsilon(x, y) \Gamma Z_v^{-1} \bar{1}_z \cdot \overline{E}_l^M(x, y) = 0 \tag{A3}$$

$$\nabla \cdot \mu(x, y) \overline{H}_t^M(x, y) - j \frac{\mu(x, y)}{R_0} \Gamma Z_v \bar{1}_z \cdot \overline{H}_l^M(x, y) = 0. \tag{A4}$$

The results in [9] can be seen as a special case ($N = 1$) of the general equations (A1)-(A4). Elimination of the longitudinal partial fields in (A1)-(A4) leads to two eigenvalue equations with the modal propagation factor matrix Γ ($=$ diag $(\gamma_p)_{[N \times N]}$) as eigenvalue and with the modal transversal fields as eigenvectors. The electric eigenvalue equation is found to be

$$[\Gamma^2 + \omega^2 \mu\epsilon] \overline{E}_t^M(x, y) = \mu \nabla \times \left[\frac{1}{\mu} \nabla \times \overline{E}_t^M(x, y) \right] - \nabla \left[\frac{1}{\epsilon} \nabla \cdot \epsilon \overline{E}_t^M(x, y) \right] \tag{A5}$$

and the magnetic eigenvalue equation is

$$[\Gamma^2 + \omega^2 \mu\epsilon] \overline{H}_t^M(x, y) = \epsilon \nabla \times \left[\frac{1}{\epsilon} \nabla \times \overline{H}_t^M(x, y) \right] - \nabla \left[\frac{1}{\mu} \nabla \cdot \mu \overline{H}_t^M(x, y) \right]. \tag{A6}$$

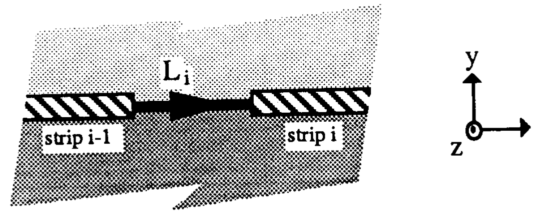

Fig. 8. Integration path relevant to the PV-formulation.

Appendix B
PV-Formulation

Both the circuit model and the real waveguide structure must have the same complex modal propagation factors γ_p and must propagate the same average complex power. However, these two fundamental conditions do not define all circuit parameters in an unique way. The remaining degree of freedom in the modeling process can be used to obtain a quasi-static equivalence.

For *slotlines and coplanar interconnection structures*, the power-voltage (PV-) model has the most TEM-like character [22] and is best suited for circuit simulation. The voltage is a physical quantity which is conserved when the structure is connected to lumped elements. The circuit voltage $V_{c_i}(i = 1, \cdots, N)$ is defined as a line integral of the electric field:

$$V_{c_i} = \int_{L_i} \overline{E}(x, y, z) \cdot d\bar{l} = \int_{L_i} V_v(z)^T \overline{E}_t^M(x, y) \cdot d\bar{l} \tag{B1}$$

with $i = 1, \cdots, N$. The voltage is a clear physical quantity that consists of contributions of the N propagating modes. The integration extends over a well-defined path (see Fig. 8). In the quasi-static limit the voltage is uniquely defined and the integration-path can be chosen arbitrary.

Equation (B1) can be written in a compact matrix-form as:

$$V_c(z) = M_v V_v(z) \tag{B2}$$

where M_v ($= [V_{ip}]_{[N \times N]}$) is the frequency dependent transformation matrix between modal voltages and circuit voltages. The ratios V_{ip}/V_{jp} $(i, j, p = 1, \cdots, N)$ are defined in an unambiguous way.

We represent the circuit current vector I_c as a superposition of the modal currents I_{v_p} $(p = 1, \cdots, N)$:

$$I_c(z) = M_I I_v(z) \tag{B3}$$

where M_I ($= [i_{ip}]_{[N \times N]}$) is the frequency dependent transformation matrix between modal currents and circuit currents.

The current components propagating in the positive z-direction can be written as:

$$I_c^+(z) = M_I I_v^+(z) = M_I Z_v^{-1} V_v^+(z) = M_I' V_v^+(z) \tag{B4}$$

where M_I' ($= [I_{ip}]_{[N \times N]} = [i_{ip}/Z_{v_p}]$) is the frequency dependent transformation matrix betwen modal voltages and circuit currents. The ratios I_{ip}/V_{jp} $(i, j, p = 1, \cdots, N)$ are uniquely defined.

Once the transformation matrices are defined, the analysis proceeds in exactly the same way as described for the PI-model.

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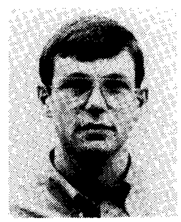

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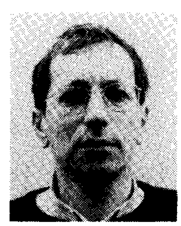

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