# Meta-modelling of microwave devices with rational functions and radial basis functions

Wouter Hendrickx[1], Tom Dhaene[1]

### Abstract

*A new versatile meta-modelling framework is presented that automatically generates accurate and compact meta-models for passive electrical components. Meta-models or surrogate models are scalable analytical models that mimic the component behaviour over a user-defined design space (consisting of frequency and layout parameters). Both multivariable rational models and radial basis function models can be used to build the meta-models.*

## 1  Introduction

CAD modelling of microwave components has been an important research topic for several years. Accurate device models are key building blocks for the synthesis and analysis of larger systems. Therefore, one wants the circuit models to be simple, accurate, compact and easy to evaluate. The last decade, a wide range of innovating techniques emerged. Several approaches for fully automated EM-based meta-model design have been developed. An automated meta-modelling framework should be a software tool, that constructs an accurate meta-model of an electrical component, without any user interaction. It should be a black-box system that builds compact, scalable meta-models for general components.

In [1, 2], the authors tackle the problem of adaptive meta-model building for waveguide applications. Using either radial basis functions (RBF) meta-models or multivariable polynomial and rational meta-models (called the Cauchy method in [1]), the authors construct meta-models for a wide range of parametrised EM-simulations.

In [3,4], the authors construct polynomial and rational meta-models for planar microwave components. This approach has been integrated in commercial EDA code [5].

Other efforts incorporate Neural Network techniques (as in [6]), Kriging meta-models (as in [7]) or hybrid techniques to construct scalable meta-models.

This paper presents the multivariable meta-modelling ($M^3$) framework, that combines different types of models and adaptive sample selection with techniques from distributed computing. The design of the framework is such that all its parts (simulator evaluation, model creation, model comparison, new sample selection, . . . ) can be replaced or fine-tuned to the end-users needs. The code is publicly available as the $M^3$-toolbox (MATLAB) [8,9]. Its key features are presented in the following section, and a demonstration on a step discontinuity in a rectangular waveguide is provided afterwards.

## 2  The $M^3$ toolbox

The $M^3$-toolbox (version 1.0) consists of three main parts: the *simulator back-end*, which is responsible for running the simulator code, the *modeller back-end*, responsible for building meta-models based on multiple data samples, and the *adaptive modelling and sampling loop*. The two back-ends are replaceable depending on the users needs, while the *adaptive modelling and sampling loop* is quite rigid in the current implementation, but will be expanded and refactored into a configurable framework in future versions of the toolbox.

### 2.1  The *simulator back-end*

This part of the toolbox is responsible for running the simulator code. The *adaptive modelling and sampling loop* can request the back-end to run simulations for the specific parameters in the design space, and the back-end will return the resulting data whenever its calculation has terminated. There are two main reasons for separating the simulator from the rest of the code.

The first reason is that different EM-solvers can be used. Some will be MATLAB-based, and can be linked to the toolbox directly, but most will be binary codes. In both cases the parameter passing to these codes may very well differ.

The second reason is that we will add a distributed computing layer to our code in version 2.0. The adaptive sampling algorithm can select multiple new data locations. All of these data points can be evaluated in parallel on a computer cluster of grid to speed-up the overall meta-modelling process.

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[1]Universiteit Antwerpen, Middelheimlaan 1, B-2020 Antwerpen, Belgium, {`wouter.hendrickx,tom.dhaene`}`@ua.ac.be`
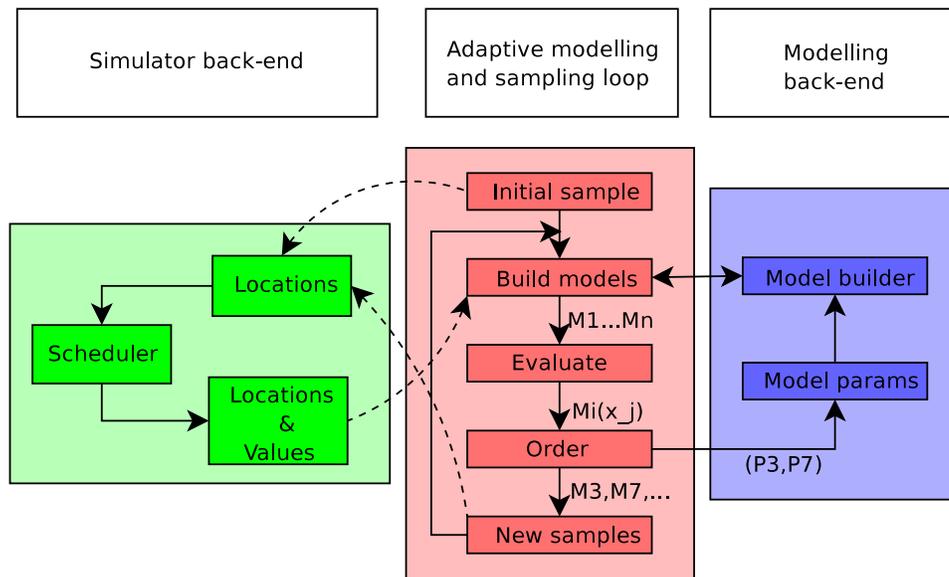
Figure 1: $M^3$ toolbox flowchart

## 2.2 The *modeller back-end*

On the other side of the *adaptive modelling and sampling loop* is the *modeller back-end*. This part is responsible for building the meta-models based on a set of samples and values, and for selecting suitable sets of meta-modelling parameters (e.g. importance of each variable).

By extracting the model building from the main loop, it is very easy to implement different modelling methodologies in the toolbox. Currently, we have a multivariable rational and polynomial modeller and a radial basis functions (RBF) modeller available within the toolbox.

Multivariable rational and polynomial models [1,3,8] are a natural choice for EM-based meta-modelling of electronic devices. Transfer functions of EM-devices can always be represented by a rational function (pole-zero) in the frequency domain. This partially explains why rational functions perform so well for the meta-modelling problems under consideration.

Radial basis function methods [10–12] are very popular for scattered data interpolation. They try to approximate any amount of data samples by a linear combination of translates of a single basis function. The main advantage, compared to polynomial models, is that they can handle huge amounts of sample points. On the other hand, computations involving RBF quickly become infeasible as the dimension of the input space increases. Unlike rational functions they lack a theoretical connection with the physical problems at hand, so one would expect less favourable results when using RBF for meta-modelling.

Other modelling techniques have been used in combination with adaptive sampling. In [7], Kriging meta-models (see [13]) are used for adaptive meta-modelling purposes. In [6], neural networks are used for EM-device modelling. In theory, all meta-modelling methodologies are applicable to the problem at hand. It is still under investigation which methods perform better.

## 2.3 The *adaptive modelling and sampling loop*

The heart of the $M^3$ toolbox is the *adaptive modelling and sampling loop*. It is responsible for deciding which samples to select in the input space and which models to build. It loops through a sequence of tasks. First of all an initial set of data samples is selected in the input space, and the *simulator back-end* is called to run the simulator for these sample points. Then several parametrised meta-models are built using the *modeller back-end*. Afterwards, these meta-models are compared in order to decide which of the meta-models are the most accurate. New simulations are requested at the locations where the most accurate meta-models differ the most, and new meta-models are built to replace the least accurate ones. The process of building and comparing meta-models and selecting new simulation locations continues until sufficient accuracy is obtained.
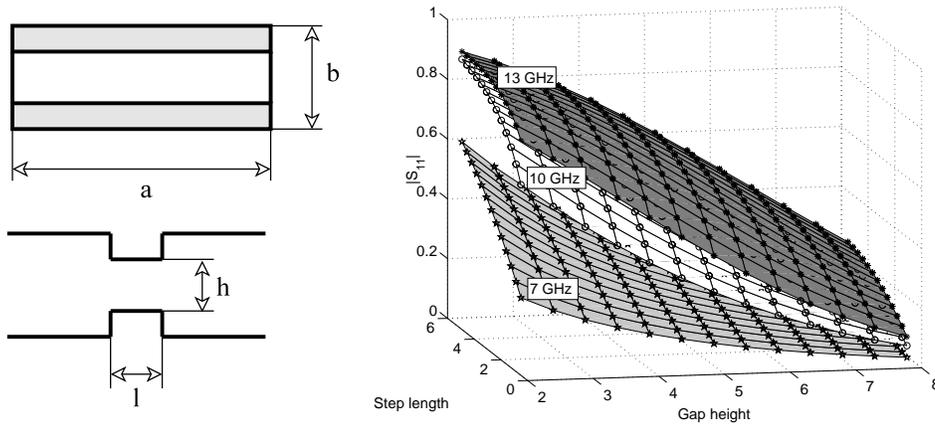
Figure 2: A step discontinuity in a rectangular waveguide (left) and plots of the $S_{11}$ scattering parameter in function of the layout parameters for three different frequencies (right)

## 3 An example problem

As an example we consider the simulation of a step discontinuity in a rectangular waveguide (See figure 2). This example has 3 inputs: frequency $f$ and the shape parameters $h$ and $l$. The outputs were the scattering parameters of the system. We used both the polynomial and rational modeller back-end and the RBF modeller back-end. Simulations were run on a single computer as the grid-based back-end is still under development [14]. For verification we ran the simulator on a dense $25^3$ grid. This is still feasible because in this case the EM-simulator [4] is relatively simple. Figure 3 shows a plot of the error as a function of the total number of data points of the *adaptive modelling and sampling loop*. The height of each shaded region corresponds to the percentage of grid points for which the absolute error is between the error bounds shown in the legend. The right plot shows the results obtained using the polynomial and rational modeller, while the left plot represent the results using Gaussian radial basis functions $\exp^{-\alpha\,x^2}$ where $\alpha$ could vary from 0.001 to 10. Note that the rational and polynomial modeller reaches -60dB quite quickly, while the RBF modeller requires significantly more data sample. This is partially due to the more local nature of the radial basis function interpolants.

## 4 Conclusion

We presented a new versatile meta-modelling framework for EM-based modelling, incorporating both multivariable rational functions and radial basis functions. It has been shown that rational functions tend to converge more quickly, partially because they adhere more to the physical behaviour of the component. Radial basis functions also produce useful results, and are worthwhile to consider in future research.

## Acknowledgements

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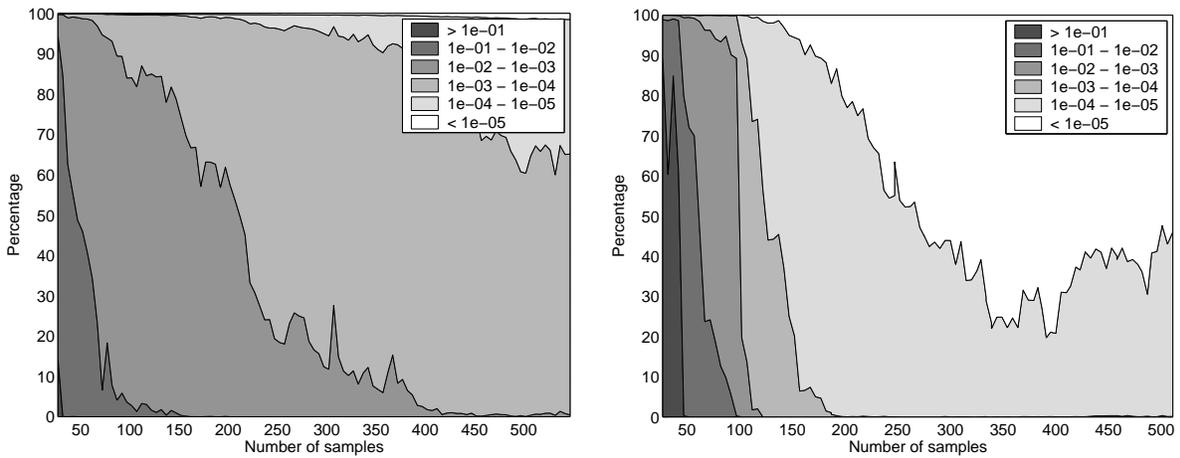
Figure 3: Convergence results using the RBF (left) and the polynomial (right) modeller

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