Accurate Macromodeling Based on Tabulated Magnitude Frequency Responses

Luciano De Tommasi[1-2], Bjørn Gustavsen[3] and Tom Dhaene[4-1]
[1]University of Antwerp, Department of Mathematics and Computer Science – Middelheimlaan 1, B-2020 Antwerp, Belgium
[2]NXP Semiconductors – High Tech Campus 37, PostBox WY4-01, NL-5656AE Eindhoven, Netherlands
[3]SINTEF Energy Research – N-7465 Trondheim, Norway
[4]Ghent University - IBBT, Depart. of Information Technology (INTEC) – Gaston Crommenlaan 8, B-9050 Ghent, Belgium
luciano.detommasi@ua.ac.be, bjorn.gustavsen@sintef.no, tom.dhaene@ugent.be

Abstract

The paper introduces an approach for rational macromodeling based on magnitude frequency data. This is achieved by fitting the magnitude square function using vector fitting (VF) with symmetrical basis functions. The procedure is demonstrated to be more robust than a direct application of VF to the magnitude square function.

Introduction

There are several situations where it is desired to model the behavior of a system with only the magnitude of the response spectrum known (e.g. [1-2]). For instance, for high power microwave devices, both magnitude and phase angle of the spectral responses can only be accurately measured with a network analyzer at relatively low frequencies. As the frequency increases, phase measurements become more and more difficult and often the spectral magnitude response may be the only available information (e.g. [2]).

This paper introduces a new procedure to identify a complex transfer function of a linear time-invariant (LTI), causal and stable system, based on a set of samples of its magnitude frequency response. The approach consists of building a rational approximation of the magnitude square response, followed by the spectral factorization of the identified rational function. The latter enables to determine a minimum-phase rational function whose magnitude approximates the given magnitude response. The basic idea of this approach was presented earlier (e.g. [3]) but its accuracy and robustness was limited due to the approximation methods available at that time. In the meantime much progress has been made in this field and this can be successfully exploited to obtain very accurate high order approximations.

The popular Vector Fitting (VF) algorithm [4-5] has already been introduced in [6] to identify high frequency models of potential transformers from magnitude data. However, complicated magnitude responses still require a more robust implementation in order to achieve very accurate approximations. The purpose of this work is to enhance the magnitude square fitting stage by introducing a modified formulation of the VF algorithm. It enforces the proper form of the rational function via symmetrical basis functions, so allowing an accurate magnitude fitting via the spectral factorization.

Magnitude Fitting via Spectral Factorization

From a mathematical point of view [7], the Laplace transform *f(s)* of the impulse response of a causal and stable LTI system is uniquely determined by the value of arg[*f*(0)] and the function:

$$\left|f(j\omega)\right|^2 = f(j\omega)\cdot f^*(j\omega)\,, \qquad (1)$$

provided that $\left|f(j\omega)\right|$ satisfies the Paley-Wiener condition:

$$\int_{-\infty}^{+\infty}\frac{\left|\ln\left|f(j\omega)\right|\right|}{1+\omega^2}\,d\omega < +\infty\,. \qquad (2)$$

In the following we assume that *f(s)* is a rational function:

$$f(s) = f_0\frac{\prod_{m=1}^{M}(s-z_m)}{\prod_{n=1}^{N}(s-p_n)}\,; \qquad (3)$$

where $M=N-1$ or $M=N$, f_0 is a real constant and both poles $\{p_n\}_{n=1..N}$ and zeros $\{z_m\}_{m=1..M}$ are either real or occur in complex conjugate pairs. The magnitude square function (1) becomes:

$$\left|f(j\omega)\right|^2 = f_0^2\frac{\prod_{m=1}^{M}\left[(j\omega-z_m)(-j\omega-z_m^*)\right]}{\prod_{n=1}^{N}\left[(j\omega-p_n)(-j\omega-p_n^*)\right]}\,. \qquad (4)$$

Since $\{p_n\}_{n=1..N}$ and $\{z_m\}_{m=1..M}$ are real or complex conjugate pairs, the (4) can be written in the form:

$$\left|f(j\omega)\right|^2 = f_0^2\frac{\prod_{m=1}^{M}\left[(j\omega-z_m)(j\omega+z_m)\right]}{\prod_{n=1}^{N}\left[(j\omega-p_n)(j\omega+p_n)\right]}\,. \qquad (5)$$

This equation suggests to approximate the magnitude square of a tabulated function $\left\{\left|\tilde{f}(j\omega_k)\right|^2\right\}_{k=1..K}$ by including both poles/zeros belonging to the left hand side of the complex plane and their symmetrical counterparts in the right hand side. Afterwards, the spectral factorization can be applied: right hand poles and zeros are discarded in order to find the corresponding minimum-phase-shift transfer function, whose magnitude approximates $\left|\tilde{f}(s)\right|$.

The VF algorithm would approximate the magnitude square response well, provided that it results in a rational expansion with opposite (symmetrical) pairs of both poles and residues (in the following $s = j\omega$):

$$\left|\tilde{f}(s)\right|^2 \cong \sum_{n=1}^{N} c_n\left(\frac{1}{s-a_n}-\frac{1}{s+a_n}\right)+d\,. \qquad (6)$$

If so, the zeros of (6) also occur in opposite pairs, as required. However, the standard VF algorithm often ends up with slightly perturbed poles and residues:

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$$\left|\tilde{f}(s)\right|^2 \cong \sum_{n=1}^{N}\left(\frac{c_n}{s-a_n}-\frac{c_n+\Delta c_n}{s+a_n+\Delta a_n}\right)+d \ . \qquad (7)$$

If the approximation (7) is fairly accurate then the perturbations Δc_n and Δa_n are necessarily small, since when $\Delta c_n \neq 0$ and $\Delta a_n \neq 0$ the right hand side of (7) has an imaginary part, whereas $\left|\tilde{f}(s)\right|^2$ is purely real. Nevertheless, we have found that rational functions like (7) give inaccurate approximations of $\left|\tilde{f}(s)\right|$. Therefore, we will in the next section enforce the required symmetry by using a proper set of basis functions.

Improving the Magnitude Square Fitting

In this section a *magnitude Vector Fitting* (magVF) algorithm is derived from VF [4]. A relaxed version of magVF was developed from [8] as well.

The application of the standard VF algorithm leads to the following linear least square (LS) problem (*pole identification* step) [4]:

$$\left(\left|\tilde{f}\right|^2 \sigma\right)_{fit}(s)-\left|\tilde{f}\right|^2 \sigma_{fit}(s) \approx 0 \ . \qquad (8)$$

Equation (8) represents the K equations obtained for different values of $s_k = j\omega_k$, where $k = 1..K$, and

$$\left(\left|\tilde{f}\right|^2 \sigma\right)_{fit}(s)=\sum_{n=1}^{2N}\left(\frac{c_n}{s-a_n^{(0)}}\right)+d \ , \qquad (9)$$

$$\sigma_{fit}(s)=\sum_{n=1}^{2N}\left(\frac{\bar{c}_n}{s-a_n^{(0)}}\right)+1 \ . \qquad (10)$$

The unknowns of (8) are $\{c_n\}$, d, $\{\bar{c}_n\}$, whereas $\{a_n^{(0)}\}$ is a set of initial poles. Once (8) is solved, a relocated and improved pole set $\{a_n^{(1)}\}$ for $\left|f\right|^2$ is given by the zeros of $\sigma_{fit}(s)$. In fact, equation (8) shows that:

$$\left|\tilde{f}\right|^2 \approx \frac{\left(\left|\tilde{f}\right|^2 \sigma\right)_{fit}}{\sigma_{fit}} \ . \qquad (11)$$

In the second step, the known poles $\{a_n^{(1)}\}$ are used to solve the following LS problem (*residues identification*):

$$\left|f\right|^2 = \sum_{n=1}^{2N}\frac{c_n^{(1)}}{s-a_n^{(1)}}+d^{(1)} \approx \left|\tilde{f}\right|^2 , \qquad (12)$$

with unknown $\{c_n^{(1)}\}$, $d^{(1)}$. The solution of (8) eventually gives the rational approximation $\left|f\right|^2$ we were looking for.

In order to enforce that poles of $\left|f\right|^2$ occur in opposite pairs, the magVF algorithm uses the pole identification scheme of (8) and (11) with the expansions (13),(14) instead of (10),(11):

$$\left(\left|\tilde{f}\right|^2 \hat{\sigma}\right)_{fit} = \sum_{n=1}^{N}\left(\frac{c_n}{s-a_n}-\frac{c_n}{s+a_n}\right)+d \ , \qquad (13)$$

$$\hat{\sigma}_{fit} = \sum_{n=1}^{N}\left(\frac{\bar{c}_n}{s-a_n}-\frac{\bar{c}_n}{s+a_n}\right)+1 \ , \qquad (14)$$

The residues identification step is replaced by:

$$\left|f\right|^2 = \sum_{n=1}^{N} c_n^{(1)}\left(\frac{1}{s-a_n^{(1)}}-\frac{1}{s+a_n^{(1)}}\right)+d^{(1)} \approx \left|\tilde{f}\right|^2 . \qquad (15)$$

Using these basis functions, mirrored along the imaginary axis, it can be verified that zeros of $\hat{\sigma}_{fit}(s)$ occur in truly opposite pairs:

$$\hat{\sigma}_{fit}(s)=\sum_{n=1}^{N}\left(\frac{2\bar{c}_n \bar{a}_n}{s^2-\bar{a}_n^2}\right)+1=\frac{\prod_{n=1}^{N}\left(s^2-\bar{z}_n^2\right)}{\prod_{n=1}^{N}\left(s^2-\bar{a}_n^2\right)} . \qquad (16)$$

The quantities $\{\bar{z}_n^2\}$ are computed as explained in [3], and the zeros $\{\bar{z}_n = a_n^{(1)}\}$ are computed as square root. Note that it may happen that $\bar{z}_n^2$ is real and negative; in that case we cannot use this as a relocated pole of $\left|f\right|^2$ because spectral factorization would lead to a single imaginary pole in $\left|f\right|$, which would be unphysical and inaccurate. In the magVF implementation, the sign of each negative real $\{\bar{z}_n^2\}$ is changed before the square root is taken. This is equivalent to change the couple of fractions:

$$\frac{j\beta}{s-j\alpha}-\frac{j\beta}{s+j\alpha} , \qquad (17)$$

which would appear in $\left|f\right|^2$, with the couple:

$$\frac{\beta'}{s-\alpha}-\frac{\beta'}{s+\alpha} , \qquad (18)$$

so leading to a single (stable) real pole in f. Though the introduced perturbation may be quite large, other poles might be relocated during the iterative process so compensating the lost contribute of $\pm j\alpha$. However, several tests have confirmed that this usually is not a problem in practice.

Examples

As a test case we consider a lossy transmission line of length 10 cm, with the following p.u.l. parameters: R = 5 Ω/m, L = 0.2 μH/m, G = 0.01 S/m, C = 300 pF/m. The *S*-parameters have been analytically computed.

Both algorithms, VF and magVF, were applied to fit $|S_{11}|^2$ and $|S_{12}|^2$, using the same number of poles and iterations. The results are respectively shown in Fig. 1 (N=20 poles, 25 iterations) and Fig. 2 (N=20 poles, 20 iterations). Both functions have been identified using unitary weights. For the $|S_{11}|^2$ response, it is seen that standard VF fails to catch the resonance peak near the 2 GHz frequency, while this does not happen with magVF. A similar result also occurs for $|S_{12}|^2$.

Figs. 3 and 4 show the absolute rms-error of VF and magVF as function of the number of iterations, respectively for $|S_{11}|^2$ and $|S_{12}|^2$. It can be seen that VF has no reliable convergence, since its rms-error does not decay monotonically as is the case for magVF.

It is worth to note that the VF algorithm may occasionally achieve a lower rms-error than magVF on the magnitude square function. This behavior is not surprising because the

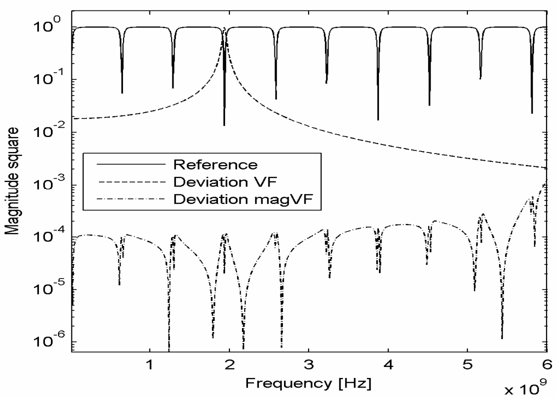

Fig. 1 Magnitude square of the reference function $S_{11}(f)$ and deviations between the reference and the identified functions via VF and magVF algorithms.

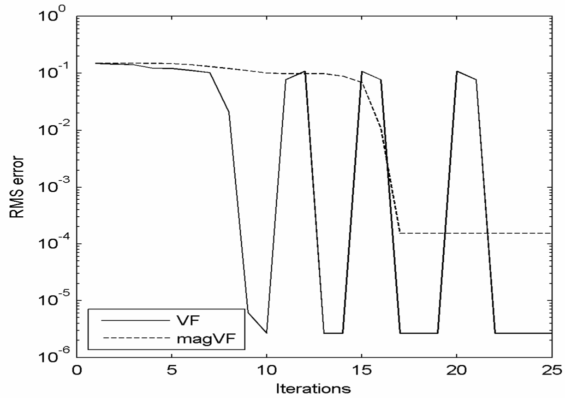

Fig. 4 Absolute rms-errors of VF and magVF as function of the number of iterations ($|S_{12}(f)|^2$ function).

VF has less constraints to satisfy than magVF, since the symmetry on poles and residues is not enforced. Nevertheless, such symmetry constraints on poles and residues are needed to recover an accurate magnitude approximation.

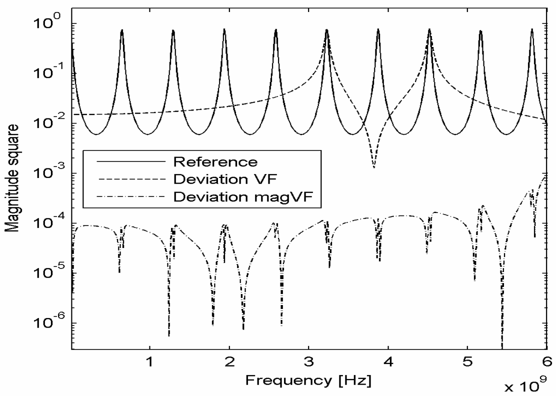

Fig. 2 Magnitude square of the reference function $S_{12}(f)$ and deviations between the reference and the identified functions via VF and magVF algorithms.

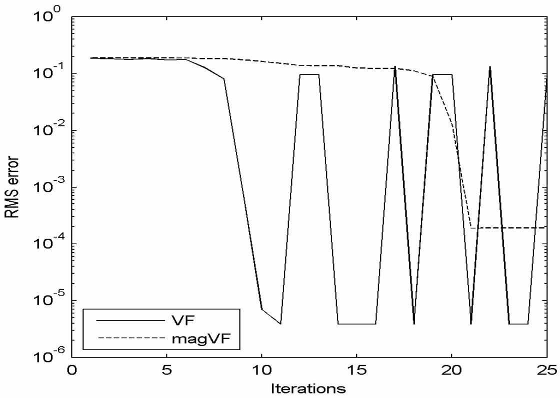

Fig. 3 Absolute rms-errors of VF and magVF as function of the number of iterations ($|S_{11}(f)|^2$ function).

Figs. 5 and 6 show the comparison between the magnitude fittings obtained with VF and magVF after the spectral factorization of the magnitude square approximations. The VF approximations shown in Figs. 5 and 6, which are highly inaccurate mostly because they do not catch the DC value, have been both obtained from accurate magnitude square approximations (10 iterations, see Figs. 3 and 4). Therefore, it is seen that an accurate magnitude square approximation is not a sufficient condition to obtain an accurate approximation of the magnitude function. In addition, it is required that poles and residues are symmetrically distributed with respect to the imaginary axis.

Discussion

A recognized weakness of the proposed approach is that the non-negative definiteness of the approximated magnitude square function $|f|^2$ in (15) has not been enforced. Such limitation arises from the usage of the unconstrained least square fitting implemented with the VF algorithm.

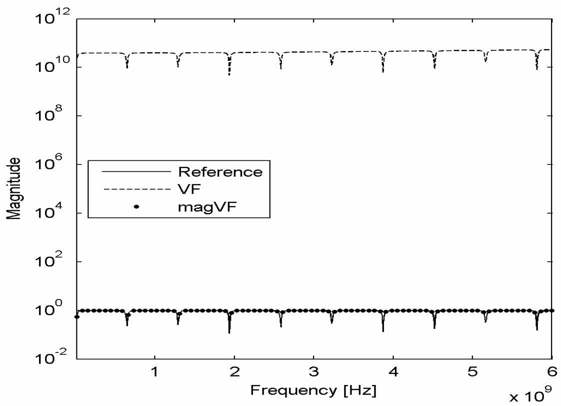

Fig. 5 Magnitude of the reference function $S_{11}(f)$ compared to those of identified functions via magVF and VF algorithms.

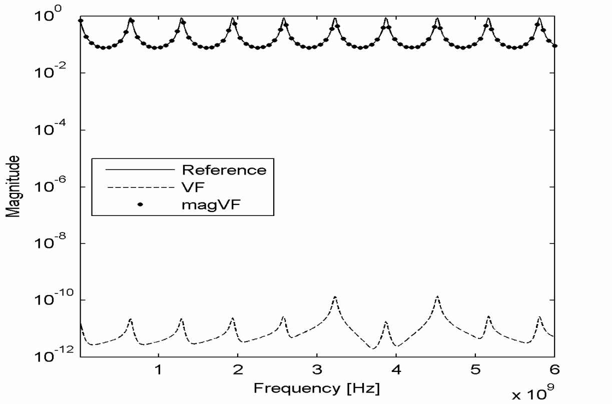

Fig. 6 Magnitude of the reference function $S_{12}(f)$ compared to those of identified functions via magVF and VF algorithms.

The non-negative definiteness property of the magnitude square rational function is a required condition to perform the following spectral factorization, which returns the magnitude approximation.

However, in most cases, the non-negative definiteness of the reference function $|f|^2$ drives the fitting process towards a rational function which has the same property. This has also happened for the two examples shown in the previous section.

Nevertheless, there are some special situations where $|f|^2$ does change its sign, becoming negative. In such cases, the function $|f|^2$ has one or more couples of imaginary conjugate zeros, located at the frequencies where the function crosses the zero line. In order to show this, we consider a rational function (19) having a single couple of such imaginary zeros $\pm j\omega_0$ (the remaining part $|f'|^2$ is assumed to have zeros and poles which are either real or complex conjugates with a non-zero real part). Now, it can be easily seen that $|f'|^2$ changes its sign at $\omega = \omega_0$. Moreover, it is worth to note that the angular frequency ω_0 may not belong to the fitting band $[\omega_{\min}, \omega_{\max}]$, therefore it would not be sufficient to enforce the non-negative definiteness of the rational approximation on the fitting band to get rid of such imaginary zeros.

$$|f|^2(j\omega) = (j\omega - j\omega_0)(j\omega + j\omega_0)|f'|^2 \approx |\tilde{f}|^2 . \qquad (19)$$

The most straightforward way to remove an imaginary conjugate couple of zeros (and so the negative values of the rational function) is to replace it with the corresponding real couple:

$$|f|^2(j\omega) = (j\omega - \omega_0)(j\omega + \omega_0)|f'|^2 \approx |\tilde{f}|^2 , \qquad (20)$$

but this results in a less accurate the approximation (of course, especially around the angular frequency ω_0).

A better solution is to somehow prevent the appearance of such negative values either by applying weights [9] or constrained optimization (to be published). The latter implementation can enforce the non-negative definiteness of the magnitude square functions over the fitting band and the asymptotic non-negative definiteness: $d > 0$ in (6).

Conclusions

The transfer function identification problem based on magnitude frequency responses can be solved by fitting the magnitude square response and then applying the spectral factorization to recover a magnitude approximation. In order to improve the robustness of the procedure, a new formulation of the Vector Fitting algorithm has been developed which uses the proper basis functions. The new approach (magVF) was demonstrated to be much more suitable for the modeling of a transmission line from S-parameters.

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