Variance Weighted Vector Fitting for Noisy Frequency Responses

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***Abstract*—This letter presents a modification of the Vector Fitting algorithm to estimate rational models of frequency responses affected by noise, e.g. in the case of measured data. It is based on the use of a least-squares weighting function that contains information about the variance of the data samples used in the macromodeling process. The modification is straightforward and does not increase the computational cost of Vector Fitting, but it significantly improves its convergence and accuracy properties in the presence of noise. The residuals test is used to assess the quality of the estimated model. Numerical results validate the proposed modification.**

***Index Terms*—Macromodeling, noise, rational approximation, vector fitting (VF).**

I. Introduction

VECTOR FITTING (VF) has become a popular tool for robust and accurate macromodeling of measured or simulated frequency responses [1]. VF has been used in different fields, such as signal integrity characterization of microwave systems [2] and Green's functions representation [3]. VF is basically a reformulation of the iterative Sanathanan-Koerner (SK) estimator [4] using partial fraction basis functions. The robustness of the method is mainly due to the use of rational bases instead of polynomials and to the relocation of the poles of these rational bases in successive iterations until the SK scheme converges. Although it is widely recognized in the microwave community that the convergence properties of VF are excellent, the convergence behavior may deteriorate when the frequency data samples contain a nonrational element such as noise, leading to a stall situation.

A relaxed non-triviality constraint was introduced in VF to improve its ability to relocate poles to better positions, thereby improving its convergence properties and reducing the significance of the choice of the initial poles [5]. This modification of standard VF is called Relaxed Vector Fitting (RVF). Another modification of standard VF, called Vector Fitting with Adding and Skimming (VF-AS), aims at addressing the convergence issues of VF when the frequency data samples are affected by noise [6]. It is based on the identification and removal of spurious poles and on an incremental pole addition and relocation process.

This letter proposes a modification of the (R)VF schemes to deal with noisy data, such as measured data. A least-squares weighting function that contains information about the measured or prior known variance of the data samples is used. The modification is straightforward and does not increase the computational cost of the (R)VF techniques, but it significantly improves the corresponding convergence and accuracy properties in the presence of noise. The new methods are called Variance Weighted Vector Fitting (VWVF) and Variance Weighted Relaxed Vector Fitting (VWRVF). A numerical example validates the new schemes.

II. Vector Fitting

We recall the VF [1] and RVF [5] algorithms to provide the background material and notation for the further developments. The goal of both methods is to approximate a frequency response $H(s)$ with an estimated rational function $R(s)$

$$H(s) \simeq R(s) = \sum_{n=1}^{N} \frac{r_n}{s - a_n} + d. \tag{1}$$

The approximation (1) is to be computed starting from a set of K frequency data samples $\{s_k, H(s_k)\}_{k=1}^{K}$. Standard VF is an iterative algorithm that refines an initial choice $\{q_n^0\}$ for the poles of the rational approximation. These initial poles are selected as complex conjugate pairs with small real parts and the imaginary parts linearly spaced over the frequency range of interest to avoid ill-conditioning [1]. Defining the following function:

$$\theta^i(s) = \sum_{n=1}^{N} \frac{c_n^i}{s - q_n^i} + 1 = \frac{\prod_{n=1}^{N} (s - z_n^i)}{\prod_{n=1}^{N} (s - q_n^i)} \tag{2}$$

where $\{q_n^i\}$ is the set of poles of the i-th iteration and $\{c_n^i\}$ are unknown residues, VF identifies the poles of the rational model $R(s)$ by solving the linear least-squares problem

$$\theta^i(s)H(s) \simeq \sum_{n=1}^{N} \frac{\tilde{c}_n^i}{s - q_n^i} + \tilde{c}_\infty^i. \tag{3}$$

The solution of (3) provides the coefficients $\{c_n^i\}$, $\{\tilde{c}_n^i\}$, $\tilde{c}_\infty^i$. Note that (3) implies that the poles of the best rational approximation of $H(s)$ must cancel out the zeros $\{z_n^i\}$ of $\theta^i(s)$. These zeros can be computed by solving an eigenvalue problem [1]

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and become the new poles $\{q_n^{(i+1)}\}$ that replace the old ones $\{q_n^i\}$. This pole relocation procedure usually converges in a few iterations (from one to three-four), leading to a final set of poles $\{a_n\}$. Once the poles $\{a_n\}$ have been identified, the terms $\{r_n\}$ and d in (1) are finally calculated by solving the corresponding least-squares problem with known poles.

In [5] the high-frequency asymptotic constraint of the function $\theta^i(s)$ in (2) is replaced with a milder summation requirement

$$\theta^i(s) = \sum_{n=1}^{N} \frac{c_n^i}{s - q_n^i} + c_\infty^i \tag{4}$$

$$\Re e\left\{\sum_{k=1}^{K}\left(\sum_{n=1}^{N} \frac{c_n^i}{s_k - q_n^i} + c_\infty^i\right)\right\} = K \tag{5}$$

where (5) enforces that the sum of the real part of $\theta^i(s)$ over the given frequency samples equals some nonzero value, without fixing any of the free variables in (4).

This relaxed non-triviality constraint may improve the ability of VF to relocate poles to better positions, thereby improving its convergence properties and reducing the significance of the choice of the initial poles [5].

III. Signal and Noise Model

In this letter, we consider that a transfer function is measured at a set of discrete frequencies. The measurement of the transfer function is assumed to be perturbed by a colored additive noise $n_H(s)$ with a zero-mean circular complex Gaussian distribution $\mathcal{N}^c(0, \sigma(s))$ [7]. The measured quantity at each sample s_k is then obtained as

$$H_m(s_k) = H_e(s_k) + n_H(s_k) \tag{6}$$

where $H_e(s)$ is the exact transfer function without the influence of the noise. One of the major advantages of frequency domain measurements is that it becomes very easy and inexpensive to measure not only the transfer function, but also the estimate of the corresponding variance [7]. To this end, it is sufficient to repeat the measurements and calculate the sample variance on a point-by-point basis. It can be shown that even for a modest number of repetitions (minimum eight), one can safely replace the exact variance by the sample variance without impairing the properties of the estimators defined in the next section [7].

IV. Variance Weighted Vector Fitting

The VW(R)VF schemes improve the convergence and accuracy properties of the (R)VF schemes in the case of noisy data, by means of a least-squares weighting function that contains information about the variance of the data samples

$$w(s) = \frac{1}{\sigma^2(s)}. \tag{7}$$

The proposed weighting function gives information to the least-squares estimator about the quality of the data samples, improving its capability of retrieving the behavior of the system under study, and reducing the disturbing effect of the noise contribution. The residuals test [7] is used to assess the quality of the estimated model, it compares the residuals $(|R(s_k) - H(s_k)|)_{k=1}^K$ with the 95% confidence bound of the data samples, in other words the model error with the level of uncertainty of the data samples. The 95% confidence bound corresponds to the $\sqrt{3}\sigma(s)$ level when a circular complex Gaussian distributed noise is assumed. A macromodeling algorithm is not able to properly describe the system dynamics contained in the data if the percentile of residuals under the defined confidence bound is not sufficiently high.

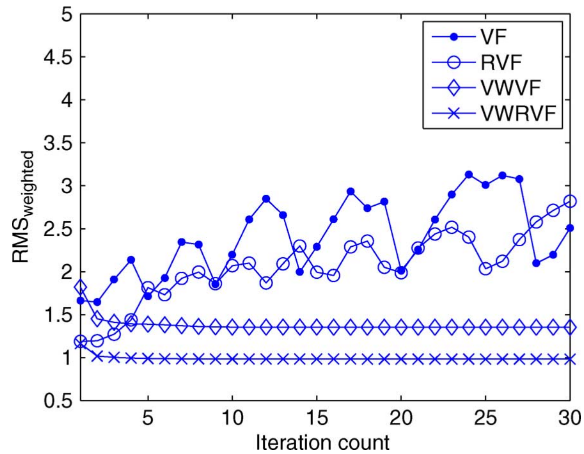

Fig. 1. Weighted RMS error as a function of the iteration number $(\mathrm{SNR} = 20\ \mathrm{dB})$.

V. Numerical Results

The proposed VW(R)VF techniques are used to model the reflection coefficient S_{11} of a microwave quarter wavelength filter over the frequency range [1–12] GHz. All data is simulated by ADS-Momentum.[1] The order of the approximation is chosen equal to $N = 30$, which leads to a maximum error of −79 dB using the RVF scheme in the case of noiseless data, and therefore to an accurate model. This data have been perturbed by a colored additive noise $n_H(s)$ with a circular complex Gaussian distribution and two different SNRs, namely 30 dB and 20 dB. The standard (R)VF schemes are compared with the new ones VW(R)VF. Fig. 1 shows the weighted root mean square (RMS) error

$$RMS_{weighted} = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\left|\frac{R(s_k) - H(s_k)}{\sigma(s_k)}\right|^2} \tag{8}$$

as a function of the iteration number for the four schemes in the case of noise with $\mathrm{SNR} = 20\ \mathrm{dB}$. It is evident from Fig. 1 that the convergence of the (R)VF schemes is impaired, while the VW(R)VF schemes do not suffer from convergence issues.

Then we have selected the models with the highest percentile of residuals under the 95% confidence bound over the 30 iterations for all four algorithms. Table I reports such percentiles, called with the acronym $GR\%$ (good residuals), for the best

[1] *Momentum EEsof EDA*, Agilent Technologies, Santa Rosa, CA.

TABLE I
PERCENTILES OF RESIDUALS UNDER THE 95% CONFIDENCE BOUND

Method	VF	RVF	VWVF	VWRVF
GR% (SNR = 30 dB)	50.7	48.6	81	95.5
GR% (SNR = 20 dB)	49.2	48	82.2	95.8

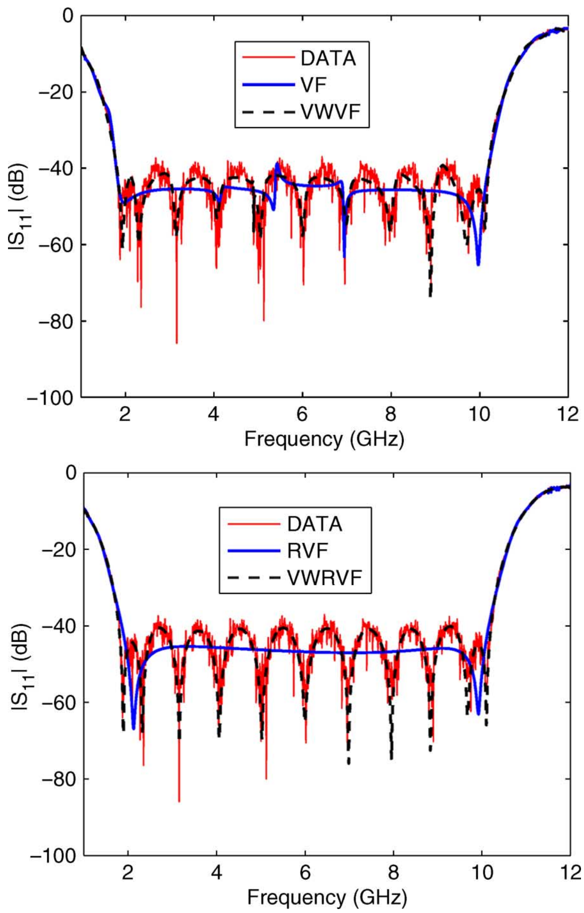

Fig. 2. Noisy data and best models $(\mathrm{SNR} = 20\ \mathrm{dB})$.

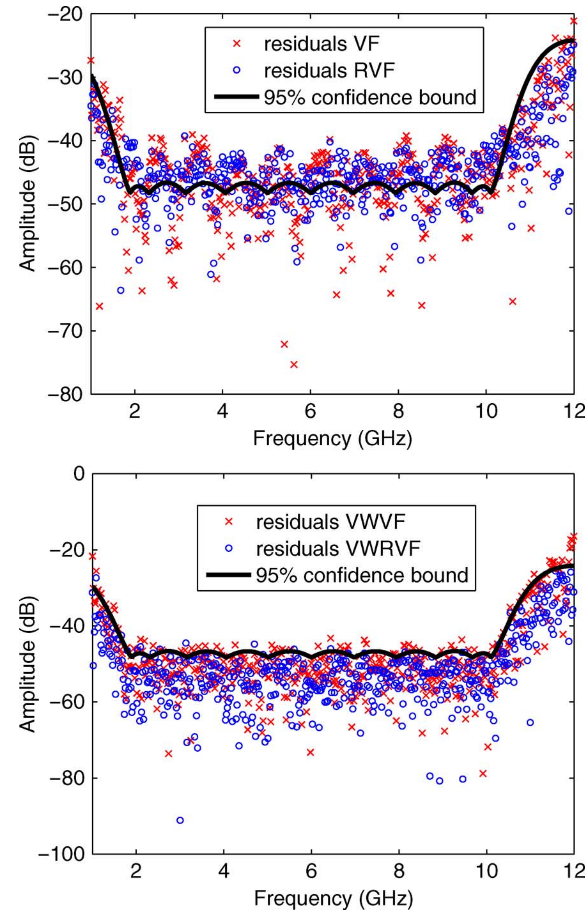

Fig. 3. Best models residuals and 95% confidence bound $(\mathrm{SNR} = 20\ \mathrm{dB})$.

models in both SNRs cases. Table I shows the capability of the proposed VW(R)VF schemes to explain very well the data, even if it is contaminated by noise.

Figs. 2, 3 compare the noisy data with the best models and the residuals of the best models with the 95% confidence bound in the case of noise with $\mathrm{SNR} = 20\ \mathrm{dB}$, confirming the table results. Fig. 2 shows that (R)VF are not able to capture the dynamics of the system under study, the noise contribution seriously damages the convergence and accuracy properties of both methods.

VI. CONCLUSION

We have presented a modification of the (R)VF schemes to deal with noisy data. It is based on a least-squares weighting function that contains information about the variance of the data samples. This modification is straightforward and does not increase the computational cost of the standard (R)VF schemes, but it significantly improves the corresponding convergence and accuracy properties in the presence of noise. A numerical example confirms the capabilities of the new VW(R)VF schemes to accurately describe the dynamics of the system under study and keep a fast and nonerratic convergence if the data is perturbed by noise. The (R)VF schemes fail in capturing the systems dynamics and show an erratic convergence behavior in the proposed example.

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