

Fast Algorithm for Broadband Macromodeling of High-Order Frequency Responses

Dirk Deschrijver ^{#1}, Bjorn Gustavsen ^{*2}, Tom Dhaene ^{#3}

[#] *Department of Information Technology (INTEC), Ghent University - IBBT
Sint-Pietersnieuwstraat 41, 9000 Ghent, Belgium*

¹ *dirk.deschrijver@intec.ugent.be*

³ *tom.dhaene@intec.ugent.be*

^{*} *SINTEF Energy Research*

Sem Saelands vei 11, 7465 Trondheim, Norway

² *bjorn.gustavsen@sintef.no*

Abstract—The calculation of broadband macromodels from large tabulated frequency responses can be computationally demanding in terms of CPU time and memory, especially in the case of high-order frequency responses. To reduce the computational burden, a fast macromodeling technique is proposed. It applies a piecewise fitting strategy that makes use of a fast rational interpolation scheme to identify a representative set of data samples and an appropriate model order. This information is exploited by the Vector Fitting algorithm to extract the poles of the macromodel in a reduced amount of time. The calculation of the residues is solved as a linear approximation problem, and standard model reduction techniques can be applied as an optional step to remove possible pole redundancies. A cable case example shows that substantial savings are obtained in terms of computation time and memory requirements.

I. INTRODUCTION

The accurate simulation of electrical interconnects at the chip, package and board level requires the usage of fast broadband macromodeling techniques. Such techniques are used for extracting frequency-dependent macromodels from measured or computed system responses in the frequency domain. The use of rational models leads to highly efficient transient simulations in the time domain, and the inclusion in circuit simulation programs (e.g. SPICE) is straightforward.

One decade ago, the well-known Vector Fitting technique was introduced in [1], which proved to be a highly robust method that is applicable to both smooth and resonant responses with high orders and wide frequency bands. This method also has the advantage that stability of the poles can be enforced using a simple pole-flipping scheme. As a result, this approach has become adopted in many societies of applied engineering, including power systems and microwave systems. Over the past few years, many extensions, improvements and enhancements were published, see e.g. [2]-[7].

Unfortunately, the calculation of accurate macromodels from highly resonant frequency responses can still be time-consuming and memory demanding. Such high-order responses occur frequently during the modeling of high-frequency/high-speed interconnection structures.

A practical solution to overcome the above problem was recently proposed in [8]. A fast rational interpolation technique

is combined with partitioning and an adaptive sampling strategy to select a reduced set of data samples that characterize the entire frequency response [9]. At the same time, a good estimate for the model order is obtained. Based on this information, the Vector Fitting technique with relaxation [2] and orthonormalization [3] is used to compute a stable pole-residue model in a significantly reduced amount of time [10].

In this paper, the power of the fast macromodeling approach is demonstrated for a difficult cable modeling example where conventional fitting methods fail. Numerical results illustrate that the method is computationally efficient and that an overall good accuracy of the macromodel is obtained.

II. OUTLINE FAST MACROMODELING ALGORITHM

The approximation of a simulated frequency response ($s, H(s)$) by a rational macromodel can be time consuming, and may require a prohibitive amount of computer resources. In order to alleviate the computational burden, a novel approach is applied that is based on partitioning of the frequency response. Although the idea of partitioning has also been considered previously to avoid ill-conditioning or to improve convergence of the fitting [11]-[14], the main motivation in this paper is to gain a speed up factor in efficiency. A full description of the algorithm is reported in [8], and it is briefly summarized below for convenience of the reader.

A. Piecewise Interpolation

Initially, the frequency range of interest [f_{\min}, f_{\max}] is divided into several neighboring partitions which are pair-wise disjoint. The width of each partition is divided in such a way that the total amount of resonant peaks is equally divided over the partitions. Each partition is treated as a separate frequency response, and it is subjected to a fast rational interpolation procedure. The algorithm selects 4 samples that are equidistantly spread over each partition, and interpolates these data samples by a Thiele-type continued fraction $R_k(s)$

$$R_k(s) = H(s_0) + \sum_{v=1}^k \frac{s - s_{v-1}}{\phi(s_0, \dots, s_{v-1}, s_v)} \quad (1)$$

where the *reciprocal* differences $\phi(s_0, \dots, s_v)$ are computed recursively from the selected data samples as in [15][16].

It is noted that this interpolation scheme is fast, since it does not require a matrix inversion or the solution of an eigenvalue problem that scales cubically with the size of the problem.

B. Sparse Sample Selection

Once the rational interpolant is constructed, the frequency response of the model is computed by a bottom-up evaluation of (1), and it is compared to the reference data at intermediate frequencies. In successive iteration steps of the algorithm, additional data samples are selected at the frequency of largest mismatch, and the interpolant is expanded in a sequential way by adding a tail to (1). This process is repeated until the interpolant approximates the response of the partition sufficiently well, up to some predefined accuracy level. Therefore, it is ensured that the model order is chosen sufficiently high to approximate the reference data, while the sparse sample distribution avoids overfitting of the data.

C. Pole Identification

Afterwards, the pole-identification step of the Vector Fitting algorithm with relaxation and orthonormalization is used to identify a set of stable poles in a robust way [10]. Based on the selected data samples and corresponding model order of the continued fraction (1), it calculates a least-squares rational function approximation $R(s)$ for each partition as follows

$$R(s) = \frac{(\sigma H)(s)}{\sigma(s)} = \frac{\sum_{p=1}^P c_p \Phi_p(s, a) + d + sh}{\tilde{c}_0 + \sum_{p=1}^P \tilde{c}_p \Phi_p(s, a)} \quad (2)$$

The basis functions $\Phi_p(s, a)$ are orthonormal rational functions which are based on a prescribed set of starting poles a . These poles are chosen according to a heuristical scheme, and they are relocated to a quasi-optimal position by solving several overdetermined least-squares problems. All the details about the Vector Fitting procedure can be found in [1].

D. Global Macromodeling

Once the previous steps are applied to each partition individually, all the poles are gathered and used to compute a global macromodel of the overall compound response. This occurs in a similar way as the residue identification step of the Vector Fitting algorithm [1]. It is noted that one could apply partitioning to the identification of residues as well, but it was found that the entire frequency response should be considered at once to obtain a robust and reliable procedure.

E. Model Order Reduction (optional)

In some cases, the global macromodel may contain some redundant poles which are not needed to have a good overall approximation, even though they are required to fit some partition of the overall response. This results from the fact that poles in the neighboring frequency ranges are neglected during the pole identification of each partition. To remove the redundancy in the model, standard model order reduction techniques can be applied [17]. It is shown in [8] that this

optional step can reduce the complexity of the model, at the expense of a small additional computational cost.

III. EXAMPLE : CABLE CASE RESPONSE

In order to demonstrate the efficiency of the macromodeling approach, an example is taken from high-voltage power systems modeling. This is a case of three parallel coaxial type cables of 1 km length. Using the nodal admittance method, a voltage transfer function is computed from a cable screen to the opposite-end phase conductor of the same cable. The resulting transfer function is shown by a blue trace in Figure 1 over the frequency range of interest 1 Hz - 10 MHz.

It can be seen that the dynamic variation of the data is quite challenging, and it is clear that a very dense sampling and a high model order is needed to capture all the sharp resonances.

The standard Vector Fitting implementation (publicly available at [18]) was used to model the data on a Pentium 4 laptop computer with a CPU clock frequency of 2.4 GHz and 2 GB RAM in MATLAB, and it was unable to compute an accurate macromodel due to a lack of memory resources.

To avoid excessive memory requirements, the novel approach in [8] is used to compute a stable pole-residue model in a reduced amount of time. The overall frequency response is divided into 40 equally spaced partitions, which are modeled with a target RMS error of approximately 1×10^{-4} (Section II-A). The data samples which are adaptively chosen by the algorithm are marked with crosses (\times), and it is seen from Figures 2 to 6 that they are not uniformly distributed over the frequency range. As desired, the sampling of the data is more dense around the resonant frequencies (Section II-B). Based on these samples and the estimated model order, the poles of each partition are extracted individually by the pole identification step of Vector Fitting (Section II-C). The total number of poles is equal to 2607, and they are gathered to compute the residues of the entire frequency response (Section II-D).

The deviation of the global macromodel is shown in Figure 1 and it is found that the root-mean-square error is equal to 3.7×10^{-4} . A closer inspection reveals that a good agreement between the macromodel and the reference data is obtained, since no visible difference can be observed. The overall computation time to calculate the fitting model takes only 19.6 seconds, which illustrates the efficiency of this approach.

It is noted that the approach can easily be extended to systems with multiple inputs and multiple outputs as in [8].

IV. CONCLUSION

Rational macromodeling of broadband frequency-domain responses becomes computationally expensive and resource demanding if the bandwidth and complexity of the structure increases. A novel approach based on partitioning, fast rational interpolation, and adaptive sample selection is applied to speed up the calculation times. A difficult cable case example illustrates that highly dynamic responses can be modelled efficiently in a significantly reduced amount of time.

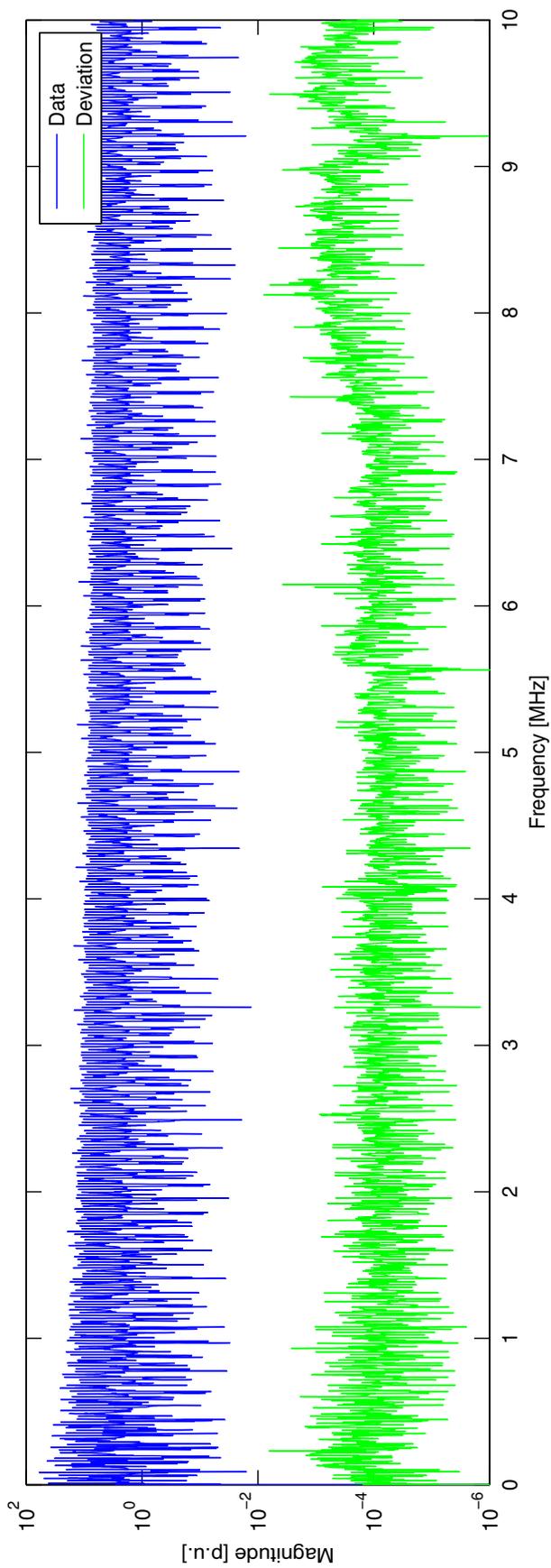


Fig. 1. Magnitude response and deviation

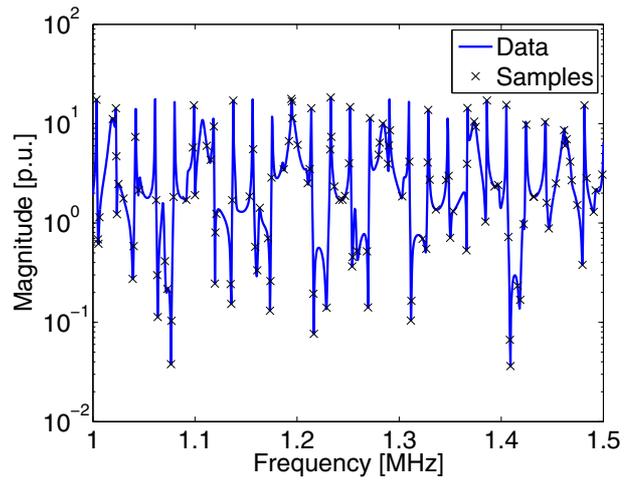


Fig. 2. Magnitude response and samples between 1 - 1.5 MHz

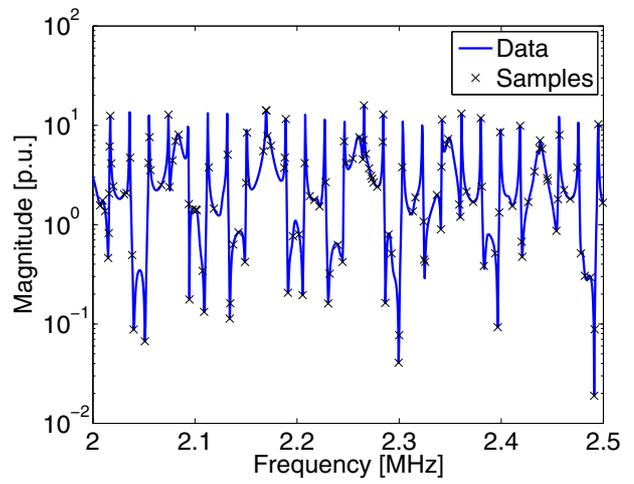


Fig. 3. Magnitude response and samples between 2 - 2.5 MHz

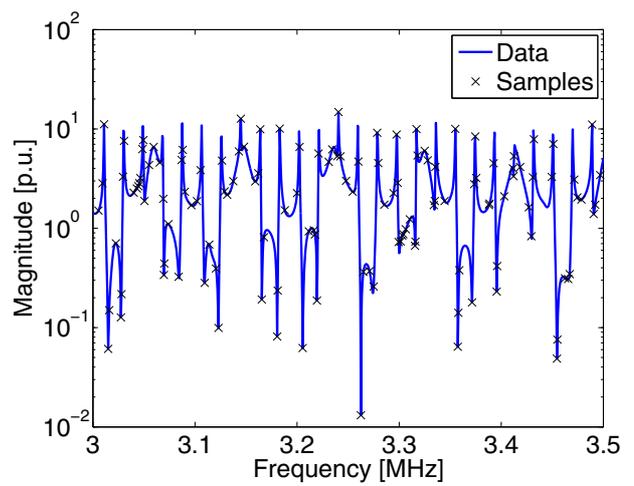


Fig. 4. Magnitude response and samples between 3 - 3.5 MHz

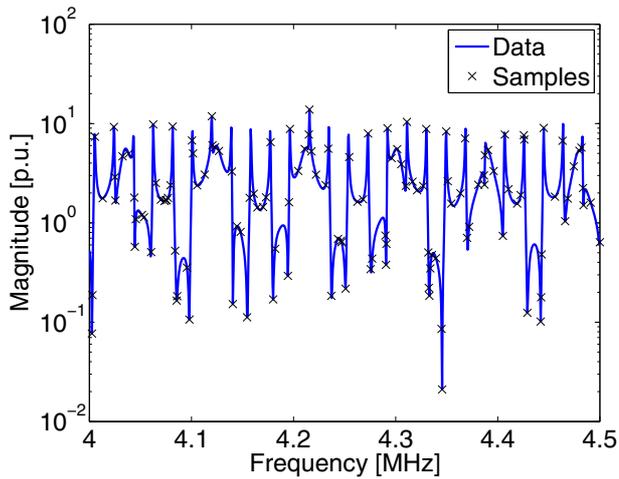


Fig. 5. Magnitude response and samples between 4 - 4.5 MHz

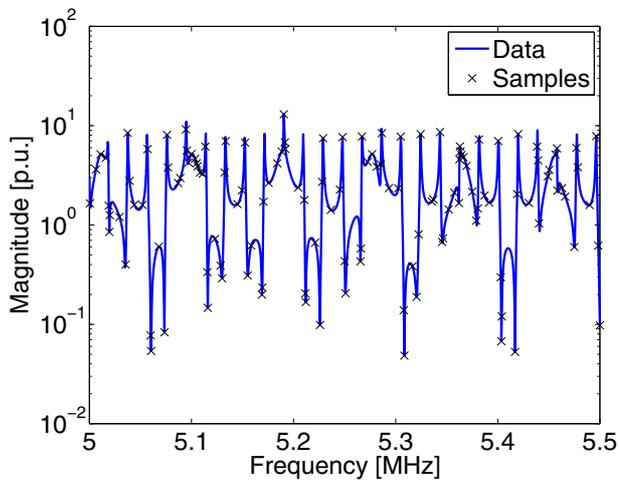


Fig. 6. Magnitude response and samples between 5 - 5.5 MHz

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