Improving robustness of vector fitting to outliers in data

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A new algorithm is introduced for broadband macromodelling of passive electronic components from frequency response data. It modifies the vector fitting algorithm in such a way that the L_1 norm of the approximation error is minimised, rather than the L_2 norm. It is shown that this approach is more robust towards outliers in the data.

Introduction: Robust broadband macromodelling techniques are of crucial importance for efficient time domain and frequency domain simulation of high-speed interconnect structures. The standard vector fitting (VF) algorithm starts from an S-parameter frequency response, sampled at a discrete set of frequencies. It computes a macromodel by minimising a weighted iterative cost function in the L_2 sense [1]. In real-life situations, the quality of the L_2 fitting model may be degraded owing to outliers in the data. An outlier is a value in the data that deviates strongly from the other values, and is usually caused by measurement or instrumentation errors. In this Letter we propose a modified vector fitting algorithm that minimises the L_1 norm of the complex fitting error instead [2]. Numerical results illustrate that the new approach is more robust with respect to outliers [3].

Vector fitting technique: Modelling algorithm: Given a discrete set of S-parameter data samples $\{s_k = j\omega_k, H(s_k)\}_{k=0}^{K}$, a rational macromodel with numerator $N^t(s)$ and denominator $D^t(s)$ can be computed iteratively ($t = 1, \ldots, T$) by successively solving least squares problems [1, 4]

$$\arg\min J = \sum_{k=0}^{K} |w(s_k)|^2 |\sigma^t(s_k)H(s_k) - (\sigma H)^t(s_k)|^2 \tag{1}$$

As in [5], both $(\sigma H)^t(s)$ and $\sigma^t(s)$ belong to a linear span of P rational basis functions $\Psi(s, a_p^{t-1}) = (s - a_p^{t-1})^{-1}$ that are based on the previously identified poles a_p^{t-1}. Note that the initial poles a_p^0 are chosen by a heuristical scheme [1]

$$(\sigma H)^t(s) = \frac{N^t(s)}{D^{t-1}(s)} = c_0^t + \sum_{p=1}^{P} c_p^t \Psi(s, a_p^{t-1}) \tag{2}$$

$$\sigma^t(s) = \frac{D^t(s)}{D^{t-1}(s)} = 1 + \sum_{p=1}^{P} \tilde{c}_p^t \Psi(s, a_p^{t-1}) \tag{3}$$

In this Letter, it is shown that the L_1 norm can be minimised by selecting the user-defined frequency-dependent weighting factor $w(s)$ in (1) as

$$w(s) = \frac{\sqrt{|H(s) - H^{t-1}(s)|}}{H(s) - H^{t-1}(s)} \tag{4}$$

where $H^{t-1}(s) = N^{t-1}/D^{t-1}$ denotes the model response at iteration step $t-1$.

Proof outline: First, define the auxiliary function $f(s)$ as

$$f(s) = \frac{|D^t(s)H(s) - N^t(s)|^2}{|D^{t-1}(s)H(s) - N^{t-1}(s)|^2} \tag{5}$$

Applying the weighting factor $w(s)$ in (4) to the cost function J in (1) yields

$$\arg\min J = \sum_{k=0}^{K} |H(s_k) - H^{t-1}(s_k)| f(s_k) \tag{6}$$

Hence, upon convergence of the iterative scheme ($D^{t-1} \to D^t$ and $N^{t-1} \to N^t$), it follows that $f(s) \to 1$. Therefore, it is clear that cost function J in (6) effectively minimises the L_1 norm of the complex fitting error $\|H(s) - H^{t-1}(s)\|_1$.

Example: Quarter wavelength filter: The reflection coefficient S_{11} of a two-port microwave quarter wavelength filter is computed over the frequency range [1–12 GHz]. Suppose that, owing to inaccuracies in the data acquisition, the S-parameter response contains six outlying data samples which are marked by black arrows. All data samples are modelled by a rational 28-pole strictly proper transfer function using the proposed fitting methodology (L_1 norm) and the standard vector fitting algorithm (L_2 norm).

The resulting macromodels are shown as a solid curve in Figs. 1 and 2, and the data samples are marked as dots. It is clear from Fig. 1 that the L_1 norm approximation yields an overall accurate result, and is not much affected by the presence of the outliers. On the other hand, the outliers lead to a strong degradation of the model quality for the L_2 norm approximation, as shown in Fig. 2. This observation is confirmed by Fig. 3 where the maximum absolute fitting error of both fitting models is shown against frequency.

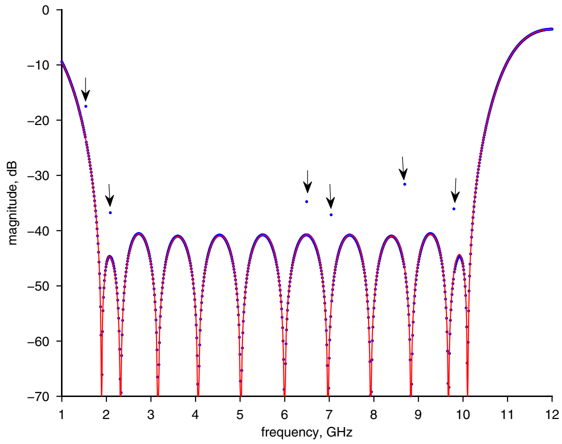

Fig. 1 *Magnitude of S_{11}: model L_1 (solid curve) and data (dots)*

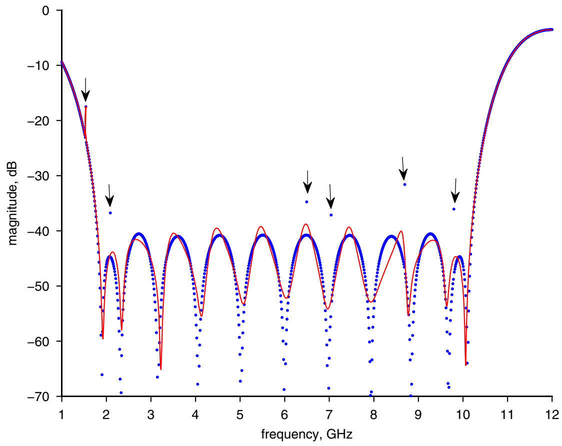

Fig. 2 *Magnitude of S_{11}: model L_2 (solid curve) and data (dots)*

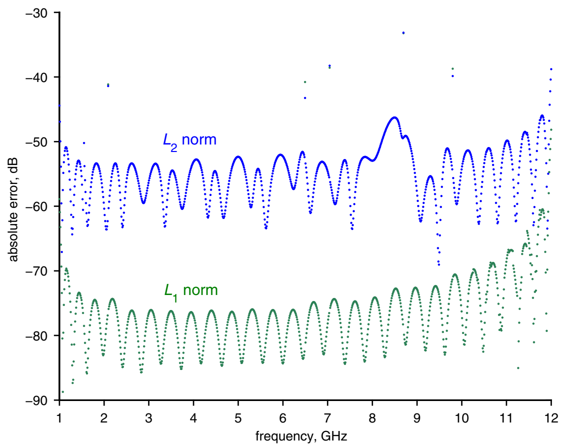

Fig. 3 *Absolute fitting error of model L_1 (bottom) and model L_2 (top)*

Conclusion: An iterative algorithm is proposed for L_1 norm identification of broadband macromodels from S-parameter data. It is shown that the method is more robust when the frequency response contains outliers. The effectiveness of the approach is illustrated by applying it to a quarter wavelength filter.

Acknowledgment: This work was supported by the Research Foundation Flanders (FWO-V).

20 May 2010
doi: 10.1049/el.2010.1364
One or more of the Figures in this Letter are available in colour online.

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