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An algorithm for direct identification of passive transfer matrices with positive real fractions via convex programming

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SUMMARY

The paper presents a new algorithm for the identification of a positive real rational transfer matrix of a multi-input–multi-output system from frequency domain data samples. It is based on the combination of least-squares pole identification by the Vector Fitting algorithm and residue identification based on frequency-independent passivity constraints by convex programming. Such an approach enables the identification of *a priori* guaranteed passive lumped models, so avoids the passivity check and subsequent (perturbative) passivity enforcement as required by most of the other available algorithms. As a case study, the algorithm is successfully applied to the macro-modeling of a twisted cable pair, and the results compared with a passive identification performed with an algorithm based on quadratic programming (QPpassive), highlighting the advantages of the proposed formulation. Copyright © 2010 John Wiley & Sons, Ltd.

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1. INTRODUCTION

The present work addresses the problem of the identification of multi-input–multi-output (MIMO) Positive Real (PR) rational transfer matrices from frequency domain data samples. Such an identification problem is motivated by the need to get reduced-order lumped approximations of complex passive electromagnetic structures, characterized either experimentally, by means of network analyzer measurements, or by electromagnetic full-wave simulations.

Reduced-order modeling is nowadays generally known as macromodeling in the area of electronic systems. This term includes either the identification of a matrix of transfer functions or the identification of a state–space realization, from data samples either in the frequency [1–8] or in the time domain [9–10]. In this work, we only consider frequency domain macromodeling.

Macromodels of passive distributed structures are building blocks of complex systems, which are usually simulated by means of circuit simulators. Owing to their mathematical representation, the inclusion of a macromodel into such circuit simulators is conceptually straightforward. Moreover, macromodeling enables the use of recursive convolutions, this way reducing the computational cost of the simulations [11].

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The identification of a reduced-order model can generally be formulated as a nonlinear constrained optimization problem. However, in order to reduce the complexity of the optimization problem, the identification schemes depart quite significantly from that general formulation, as we discuss in the following.

Electromagnetic systems subjected to macromodeling are often passive (i.e. they cannot generate energy). Relevant examples are in the area of electronic interconnects [9] or power systems [12]. Passivity is an important property, which must be preserved when generating macromodels for simulation: this means that a reduced model of a passive system should be passive as well. In fact, stable models that do not preserve passivity may lead to an overall unstable network when coupled with other (stable and even passive) circuit blocks. The instability of the overall network is not trivial to predict; therefore, macromodeling approaches must include a suitable method to achieve the passivity of the generated models.

The Vector Fitting (VF) algorithm [1, 7, 13, 14] is nowadays recognized as one of the most efficient and robust approaches for the frequency domain rational approximation. At the same time it cannot guarantee that the identified transfer function is a PR function (i.e. it represents a passive system), even if the original data set does verify passivity constraint at any frequency. Therefore, in this scheme, passivity must be checked after the model identification and, in case of any violation is found, it must be enforced by applying some post-identification perturbations to model parameters.

Passivity enforcement schemes can be classified into two groups. The first one includes those algorithms that can enforce the passivity constraints only at discrete frequency samples (e.g. [15–17]). The second one includes algorithms that can rigorously guarantee passivity at any frequency by means of Hamiltonian eigenvalue perturbation (e.g. [18–21]). All the passivity enforcement algorithms may lead to severe accuracy losses and sometimes fail to converge. On the positive side, they are applicable to large structures and scale favorably with the model complexity [22].

An alternative approach to passivity enforcement is the direct identification of a passive macromodel from data samples [23–25]. These methods enable the identification of *a priori* guaranteed passive models, which means that the passivity check and subsequent enforcement via perturbation of model parameters are not necessary. Within this category some schemes have been proposed [3–5, 26] which is based on the idea of identifying the reduced model as a combination of guaranteed passive sub-systems.

A more general and rigorous approach was presented in [23], and is based on the idea of exploiting Positive Real Lemma (PRL) [27] for *a priori* imposing passivity constraints. The identification process is divided into two steps: first the system poles are identified, then the identification of residues (with fixed poles) under the passivity constraints is formulated as a convex programming problem. This latter approach has the fundamental advantage of being theoretically sound, since the final result is the best approximation of the data set for a given pole set. The main limitation is the increased computational effort due to the convex programming formulation, which suggests limiting its use to relatively moderate complexity structures [22].

In this paper, we present an *a priori* passive identification algorithm based on convex programming, namely the 'Positive Fraction Vector Fitting' (PFVF). The PFVF algorithm was first introduced in [24] for single-input–single-output systems, and is extended in this paper to MIMO systems. It is basically based on the idea of expanding the transfer matrix in a pole-residue form enforcing that all the single 'fractions' are PR functions. This sub-optimal formulation (as compared with [23]) aims at reducing the computational effort while preserving the rigorous enforcement of passivity constraints via convex optimization, at the same time generalizing the approach of constraining passivity in sub-system previously mentioned.

After a discussion of the main features of the proposed formulation contrasted to the general PRL approach, we first describe the algorithm implementation, then we validate and test it on the identification of a reduced passive model for a typical unshielded twisted pair (UTP) cable. Some practical advantages of the proposed formulation are then brought to evidence by the comparison to a well-established passivity enforcement algorithm [15].

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2. GENERAL CONDITIONS FOR PASSIVITY IN CIRCUITS

Admittance and impedance matrices of passive electrical networks (hybrid representation) are PR matrix rational functions [28]. A square matrix $H(S)$ is said to be PR if it satisfies the following conditions:

$$H(s)\ \text{is analytic} \quad \text{for}\ \mathrm{Re}\{s\}>0, \tag{1}$$

$$H^*(s) = H(s^*) \quad \text{for}\ \mathrm{Re}\{s\}>0, \tag{2}$$

$$H(s) + H^H(s) \geqslant 0 \quad \text{for}\ \mathrm{Re}\{s\}>0, \tag{3}$$

where $*$ indicates the complex conjugate, H the Hermitian (conjugate transpose), and $\geqslant$ the positive semi-definiteness.

When a scattering representation is used the positive realness condition (3) is replaced by the bounded real condition:

$$I - H^H(s)H(s) \geqslant 0 \quad \text{for}\ \mathrm{Re}(s)>0. \tag{4}$$

In the present work only hybrid representation is considered. However, this does not result in a loss of generality because of the practical equivalence of the two representations.

Note that conditions (1) and (2) are satisfied by any strictly stable transfer functions (i.e. a rational function of s with all its poles in the left half plane); therefore, only (3) has to be checked/enforced in practice when dealing with any approximation that involves stable poles. Condition (3) is equivalent to:

$$\mathrm{eig}\{\mathrm{Re}[H(\mathrm{j}\omega)]\} \geqslant 0 \quad \text{for any}\ \omega>0, \tag{5}$$

where 'eig' denotes the entire set of the eigenvalues of $H(\mathrm{j}\omega)$.

The conditions (1)–(3) can be expressed in an equivalent form basing on a state–space realization of the considered transfer function:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = Ax(t) + Bu(t), \tag{6}$$

$$y(t) = Cx(t) + Du(t), \tag{7}$$

where n is the system order, m is the number of inputs and outputs, $A \in \Re^{n\times n}$ is the state matrix, $B \in \Re^{n\times m}$ is the input matrix, $C \in \Re^{m\times n}$ is the output matrix, and $D \in \Re^{m\times m}$ is the direct term. Positive realness of the transfer function $H(s) = D + C(sI - A)^{-1}B$ is equivalently expressed by the following Positive Real Lemma (PRL) [27]:

Theorem

Let $\{A, B, C, D\}$ be a controllable state–space model whose transfer function is $H(s) = D + C(sI - A)^{-1}B$. Let $\{A, B, C, D\}$ be stable, i.e. all of poles of $H(s)$ are either in the left half plane or on the imaginary axis, in which case they are simple. If there exists a $K = K^{\mathrm{T}}$ such that the following linear matrix inequalities are satisfied:

$$\begin{bmatrix} -A^{\mathrm{T}}K - KA & -KB + C^{\mathrm{T}} \\ -B^{\mathrm{T}}K + C & D + D^{\mathrm{T}} \end{bmatrix} \geqslant 0, \qquad K \geqslant 0, \tag{8}$$

then $H(s)$ is PR. Vice-versa, if $H(s)$ is PR then a matrix $K = K^{\mathrm{T}}$ exists such that (8) are satisfied.

3. PASSIVE IDENTIFICATION VIA CONVEX PROGRAMMING AND PFVF

Condition (5) is often used for the passivity enforcement on a discrete set of frequencies $\{\omega_i\}$, $i \in I$ chosen in the frequency intervals where at least one eigenvalue assumes negative values (passivity violation intervals). Although this approach can work well in several applications [15–17], further research has been overtaken to rigorously guarantee the passivity at any frequency [18, 19, 23, 24]. Nevertheless, to the best of our knowledge, all the passivity

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enforcement methods are iterative algorithms to a certain extent based on linearization. They iteratively apply first-order perturbations to (non-passive) model parameters for correcting passivity violations. Several iterations may be required to converge, and even the convergence may not be guaranteed every time (i.e. passivity violations do not disappear).

An *a priori* passive identification approach based on convex programming has been introduced in [23] by Coelho *et al*. The possibility of formulating a convex minimization problem follows the idea of a two-step identification process, where the poles are identified first. Then the PR constraints (8) become convex (positive) semidefinite constraints by keeping the matrices A and B as constants in the optimization problem. In this way, products of variables (matrices) do not appear in the optimization problem. Matrix A may be pre-estimated by means of a pole identification algorithm, without taking into account the passivity constraints. Matrix B does not need to be estimated, since it can be kept fixed exploiting the degrees of freedom available in the state–space representation (see [23] for details). Hence, assuming a convex error function (to be minimized), convex programming can be applied. Matrices C and D can be computed by minimizing the approximation error, while requiring that the PRL is satisfied. Assuming that all the frequency samples $\{\tilde{H}(\mathrm{j}\omega_k)\}_{k=1\ldots K}$ equally contribute to the least-squares error (no different weights applied), the identification problem can be settled as

$$
\begin{aligned}
&\{C,D\} = \underset{C,D,K}{\operatorname{argmin}} \left\| \operatorname{vec}\left(H(\mathrm{j}\omega_k) - \tilde{H}(\mathrm{j}\omega_k)\right) \right\|_2 \\
&\text{subject to :} \\
&\begin{bmatrix} -A^{\mathrm{T}}K - KA & -KB + C^{\mathrm{T}} \\ -B^{\mathrm{T}}K + C & D + D^{\mathrm{T}} \end{bmatrix} \geqslant 0, \\
&K \geqslant 0.
\end{aligned} \tag{9}
$$

where $H(\mathrm{j}\omega) = D + C(\mathrm{j}\omega I - A)^{-1}B$ is the identified transfer function. Although this approach is mathematically optimal (being based on a necessary and sufficient condition), the optimization algorithm has to handle more variables than the model parameters (matrices C and D), since the matrix K is a part of the optimization problem.

The PFVF identification procedure works as follows. Let us consider a pole residue expansion of the transfer matrix as

$$
H(s) = R_0 + \sum_{i=1}^{N} \frac{R_i}{s - p_i}, \tag{10}
$$

where each term is subject to the passivity constraints (5). It is trivial to determine that for the R_0 term, and for the case of $R_i/(s-p_i)$ with p_i a real pole, condition (5) in terms of the residue matrices rewrites:

$$
\begin{aligned}
R_0 &\geqslant 0, \\
R_i &\geqslant 0.
\end{aligned} \tag{11}
$$

The case of complex conjugate poles, once the pairs $(R_i/(s-p_i)) + (R_i^*/(s-p_i^*))$ are considered together, leads to the conditions for the residue matrices R_i:

$$
\begin{aligned}
&-[\mathrm{Re}(p_i)\mathrm{Re}(R_i) + \mathrm{Im}(p_i)\mathrm{Im}(R_i)] \geqslant 0 \\
&-[\mathrm{Re}(p_i)\mathrm{Re}(R_i) - \mathrm{Im}(p_i)\mathrm{Im}(R_i)] \geqslant 0
\end{aligned} \tag{12}
$$

Note that:

1. constraints (11)–(12) are frequency independent;
2. after the poles have been identified, the constraints expressed by (12) are linear matrix inequalities with respect to the residues.

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For any approximation of an immittance matrix in the form (10) under the constraints (11)–(12), the passivity of the whole expansion follows trivially from that of each term. Moreover, it is important to note that such passivity conditions are sufficient but not necessary, since interconnection of passive and active systems (circuits) may lead to passive systems as well. This may lead in principle to sub-optimal solutions as compared with the PRL approach.

As we already mentioned, the idea of exploiting passive 'building blocks' (named 'Positive Fractions' (PF) in this work) to realize passive macromodels is common also to other identification methodologies, as in [4–5]. The novel contribution of PFVF is the identification scheme, which combines VF pole identification with convex identification of residues.

After the identification problem is formulated as above, we now describe the identification procedure we settled. First, the (stable) poles estimation is pursued by means of standard VF. Identification of residues $\{R_i\}$ in (10) is pursued by minimizing the error between data samples $\{\tilde{H}(\mathrm{j}\omega_k)\}_{k=1..K}$ and the approximation model (evaluated in the same frequency samples) $\{H(\mathrm{j}\omega_k)\}_{k=1..K}$, under the passivity constraints (11)–(12), formulating the constrained norm minimization problem:[‡]

$$
\begin{aligned}
&\{R_i\} = \underset{\{R_i\}}{\operatorname{argmin}} \left\| \operatorname{vec}\left(H(\mathrm{j}\omega_k) - \tilde{H}(\mathrm{j}\omega_k)\right) \right\|_2 \\
&\text{subject to :} \\
&R_0 \geqslant 0 \\
&\text{for } i = 1 : N \\
&\quad \text{if } p_i \text{ is real} \\
&\qquad R_i \geqslant 0 \\
&\quad \text{else if } p_i \textit{ and } p_{i+1} \text{ are a complex conjugate pair} \\
&\qquad\quad -[\operatorname{Re}(p_i)\operatorname{Re}(R_i) + \operatorname{Im}(p_i)\operatorname{Im}(R_i)] \geqslant 0 \\
&\qquad\quad -[\operatorname{Re}(p_i)\operatorname{Re}(R_i) - \operatorname{Im}(p_i)\operatorname{Im}(R_i)] \geqslant 0 \\
&\text{end}
\end{aligned}
\tag{13}
$$

The problem (13) is convex (e.g. [29]). In order to solve it, we use CVX [30, 31], a package for specifying and solving convex programs. CVX accepts a convex program specification in the form (13), automatically transforms it into a semidefinite program, and finally calls the relevant solver (SDPT3 or SEDUMI). It is worth to note that a semidefinite programming solver does not accept a norm minimization problem, since the standard form of a semidefinite program involves the minimization of an affine function [32].

We conclude this section with some remarks about the computational cost of the considered formulation. There is a certain trade off between perturbative algorithms and convex programming-based algorithms: with the former convergence to passive models is not *a priori* guaranteed, and the identified model can degrade significantly the accuracy; on the other hand computational cost increases slower as compared with convex programming [22]. Therefore, the convex programming approach, although recognized as more general and theoretically sound, is suggested for problems of moderate complexities (number of ports $\times$ number of poles). What is to be considered specifically for PFVF convex formulation is that the number of unknowns of the optimization grows linearly with the model order n, whereas with the convex formulation based on PRL [23], it grows quadratically (due to the presence of the $(n^2+n)/2$ additional variables of auxiliary matrix K). In particular, it can be easily verified that the optimization problem (9) has $mn+m^2+(n^2+n)/2$ unknowns, whereas the problem (13) has just $(N+1)m^2$ unknowns, with $n=mN$ [33], where the number of unknowns can be reduced of about half by enforcing symmetry of H in proper cases.

[‡]The vec$(\cdot)$ operator stacks the column of the matrix into a single vector, e.g. $\operatorname{vec}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{12} \\ a_{22} \end{pmatrix}$.

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4. A VALIDATION CASE: MACROMODELING OF TWISTED CABLES

In this section, the proposed PFVF approach is validated with reference to the identification of a passive macromodel of a typical Unshielded Twisted Pair (UTP) as shown in Figure 1. Furthermore, the same case study is also tackled by means of a highly effective passivity enforcement procedure (QPpassive [15]), and results obtained with the two algorithms are compared.

Twisted cables are largely used to reduce EMI effects induced by external fields and crosstalk produced by parallel wires. In the past, they were modeled by means of the transmission line model to predict susceptibility and crosstalk. However, the radiation and dispersion effects cannot be adequately taken into account through this model; hence, giving an incorrect description of the high-frequency behavior [34]. A more accurate characterization can be achieved by full-wave electromagnetic simulations [35]. The typical outcome of such simulations are admittance matrix frequency samples, which can be used to derive a macromodel. In [34] the passivity issue is not considered at all, leading to non-passive macromodel with the already mentioned negative consequences.

We consider as case study two twisted copper wires interconnect, with a twist pitch of 16.95 mm, a circular cross section of radius 0.25 mm, and a center-to-center distance between the conductors of 0.9 mm. Three twist pitches of this interconnect structure have been simulated with a full-wave code named SURFCODE, which implements the approach [35]. Admittance matrix frequency samples were computed in the frequency interval [0–1 GHz] (100 linearly spaced samples) and used for the identification of a passive reduced-order model.

The identification of a reduced model for such a structure performed with the PFVF algorithm is compared with the quadratic programming passivity enforcement algorithm (QPpassive) introduced in [15]. Note that the same macromodeling problem was tackled in [6] with the approach [18], which is not therefore considered in this paper.

Figures 2 and 3 show, respectively, the reference (data) of Y_{11}, Y_{12} (magnitude and phase) vs identification as obtained with the proposed PFVF algorithm, standard VF, and VF followed by the QP-passivity enforcement (with a single or more iterations). The comparison has been performed with the same number of poles ($N = 16$) and the same number of VF iterations (10).

As readable from Figures 2 and 3, all the considered identification schemes achieve similar and satisfactory accuracies. The model generated with pure VF identification [1] exhibits passivity violations outside the band of the frequency response used for identification ([0–1 GHz]). In particular, the first eigenvalue has a large negative spike between 1.5 and 2 GHz (Figure 4(a)), whereas the second one is always positive (Figure 4(b)). We just note that passivity violations can originate unstable transient simulations even when they arise outside the frequency band of interest, since PR part poles may appear and be excited (e.g. [5]).

The QPpassive algorithm enforces passivity at a discrete set of frequencies where violations occur, the number and location of such samples being user-defined parameters. In the shown example, a first attempt to compensate the passivity violation was performed by enforcing the passivity constraint at the frequency sample corresponding to the minimum of the first eigenvalue (Figure 4(a, b): VF+QP no iterations). Although greatly damped, the violation in the first eigenvalue still remains (Figure 4(a)). In addition, a new violation appears on the second

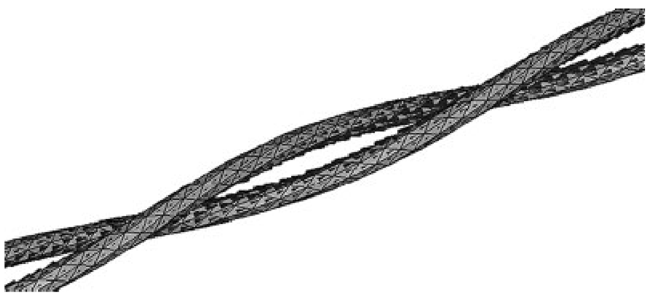

Figure 1. Schematics of a two-conductors unshielded twisted cable with a typical mesh for SURFCODE simulation.

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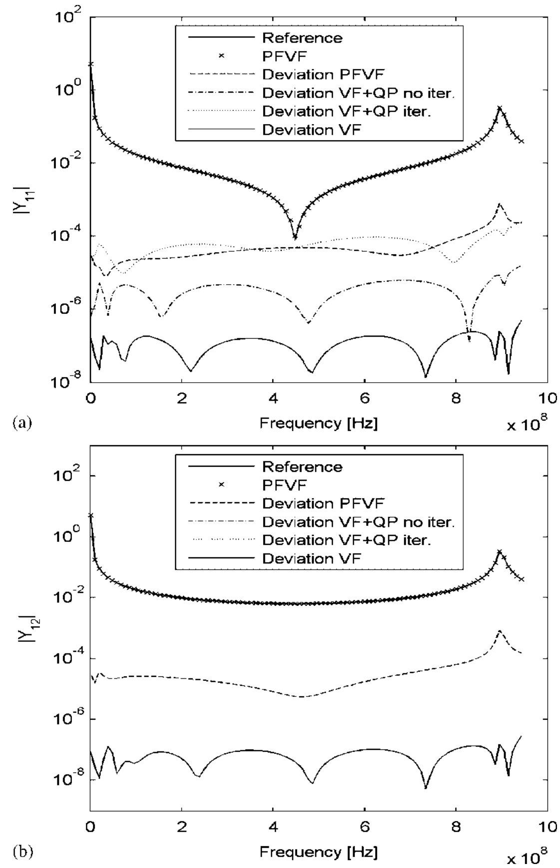

Figure 2. (a) Magnitude of the admittance functions Y_{11} of a two-conductors twisted cable compared with its VF, PFVF and VF+QPpassive approximations and (b) Magnitude of the admittance functions Y_{12} of a two-conductors twisted cable compared with its VF, PFVF and VF+QPpassive approximations. The three curves 'Deviation VF', 'Deviation VF + QP no iter.', 'Deviation VF+QP iter.' are overlapped because QPpassive did not modify the Y_{12} term.

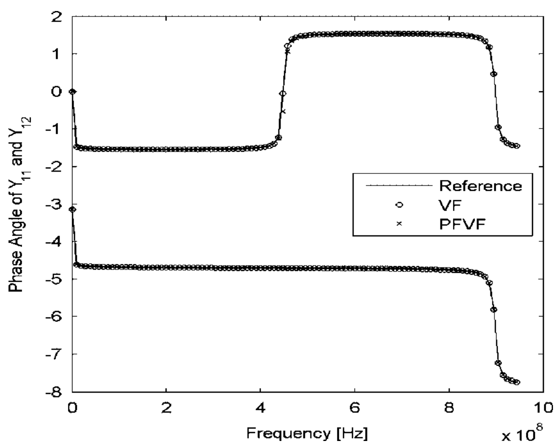

Figure 3. Phase angle of the admittance functions Y_{11}, Y_{12} of a two-conductors twisted cable compared with VF and PFVF approximations.

eigenvalue (Figure 4(b)). This unsuccessful attempt suggests implementing an iterative scheme, where a new QP-enforcement occurs until all the passivity violations are removed (Figures 2, 4: VF+QP iterations). A new QP-iteration has to enforce that the model is passive at those frequencies where violations arose in the previous iterations (in addition to frequencies where

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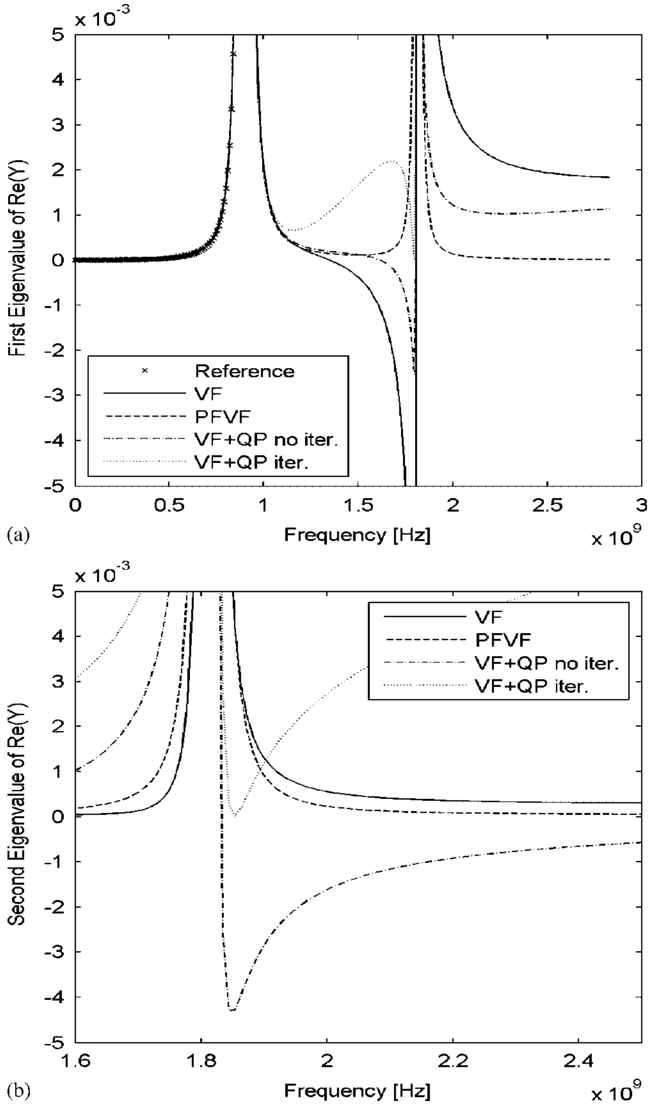

Figure 4. (a) First eigenvalue of the 2×2 matrix Re(Y) of a two-conductors twisted cable compared with VF, PFVF and VF+QPpassive approximations and (b) Second eigenvalue of the 2×2 matrix Re(Y) of a two-conductors twisted cable compared with VF, PFVF and VF+QPpassive approximations.

violations of the original model were located). Several variants are possible, and it is up to the user to implement the strategy that works better for the problem at hand [36]. Furthermore, passivity cannot be guaranteed, however it is often achieved. This also happens in the given example. Note that the QP-passivity enforcement was performed modifying only the diagonal elements of the residue matrices. This can be seen from Figure 2(a, b), which show that the error (deviation with respect to the reference data) of the VF approximation of Y_{11} is increased after the passivity enforcement, whereas the error of Y_{12} remains the same. A more accurate result could be expected when more free variables are included in the optimization problem, but this increases the computational cost.

The results shown in this section are summarized as follows. Both the PFVF and VF+QPpassive algorithms successfully achieved the identification of a passive macromodel of the considered case study, with similar accuracies. The proposed PFVF algorithm solves the identification problem by means of a single (convex) optimization problem. On the other hand, with the QPpassive algorithm, as the enforcement of passivity at some frequency locations may fail or result in new passivity violations at some other frequencies, the user has to implement a

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proper iterative strategy. This may be done in different ways and typically involves the solution of several quadratic programming optimization problems.

5. CONCLUSIONS

We introduced a new algorithm for the direct identification of passive MIMO macromodels from frequency response data samples. Passivity constraints are imposed on the single transfer function terms via convex programming. This sub-optimal formulation reduces the number of optimization variables with respect to the optimal identification scheme based on the PRL. The new approach has been successfully validated on a twisted cable macromodeling problem and compared with an iterative passivity enforcement algorithm (QPpassive). Major advantages of the proposed formulation are:

(1) model passivity is *a priori* guaranteed at any frequency (as passivity constraints are frequency independent); therefore, the search for passivity violations and subsequent enforcement are not necessary;
(2) if the convex optimization problem is feasible then the passivity is achieved by solving a single optimization problem, whereas passivity enforcement algorithms require the iterative solution of optimization problems;
(3) the complexity of the convex optimization problem is reduced with respect to the general formulation [23] based on the PRL, as the number of variables grows linearly with the model order instead of quadratically.

APPENDIX A: CONVEX OPTIMIZATION PROBLEMS

The general form of a (non-linear) optimization problem is:

$$\begin{aligned}&\text{minimize } f_0(x)\\&\text{subject to :}\\&f_i(x)\geqslant 0,\quad i=1\ldots m,\\&h_i(x)=0,\quad i=1\ldots p.\end{aligned}\tag{A1}$$

An optimization problem is said to be 'convex', when f_0 and the inequality constraints f_i are convex functions,[§] and the equality constraints h_i are affine $h_i = a_i^{\mathrm{T}}x - b_i$ [32, 37]. Three important properties of convex optimization problems are [38]:

(1) optima are guaranteed to be global: if a local optimum is found, then the optimum is global;
(2) efficient numerical methods are available;
(3) numerical algorithms can effectively detect infeasibility, unboundedness and near-optimality (using duality theory).

Convex programming solvers are designed to handle specific problems known as *standard forms*, rather than the general form (A1). Therefore, in order to be solved, a generic convex programming problem has to be transformed into one standard form. Such transformations rely on a set of known mathematical rules. However, existing optimization software provides a high-level interface with solvers, automatically performing transformations needed to solve many convex problems formulated in non-standard forms (e.g. [31]).

[§]A function f: $\Re^n \to \Re^n \cup +\infty$ is defined 'convex' when satisfies: $f(\alpha x + (1-\alpha)y) \leqslant \alpha f(x) + (1-\alpha) f(y) \quad \forall x, y \in \Re^n, \ \alpha \in (0,1)$.

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APPENDIX B: THE QP-PASSIVE ALGORITHM

QPpassive tries to enforce the passivity conditions (3b) at a discrete set of frequencies $\{\omega_i\}$, $i \in I$ by adding a corrective term ΔH (B1) to the given non-passive model H (10):

$$\Delta H = \Delta R_0 + \sum_{n=1}^{N} \frac{\Delta R_n}{s - p_n}. \tag{B1}$$

The first-order perturbations $\{\Delta R_n\}$ minimize the change ΔH to the original model H in the least-squares sense, while enforcing the constraints:

$$\lambda + \Delta\lambda = \mathrm{eig}(H(\mathrm{j}\omega_i) + \Delta H(\mathrm{j}\omega_i)) \geqslant 0, \quad i = 1 \ldots I. \tag{B2}$$

The relation between transfer function residues and eigenvalues (3b) is linearized:

$$\Delta\lambda = R\Delta x \tag{B3}$$

being Δx a vector holding the variables of $\{\Delta R_n\}$, and the solution of the least-squares problem is achieved by solving a quadratic programming problem in the form:

$$\begin{gathered} \min_{\Delta x} \frac{1}{2}(\Delta x^{\mathrm{T}} M^{\mathrm{T}} M \Delta x) \\ \text{subject to} \quad N\Delta x < p \end{gathered} \tag{B4}$$

The details about the computation of M, N and p can be found in [15]. Finally, we remark that the problem (B4) is formulated by assuming that the relation between eigenvalue and residue perturbations is linear (B3), but this is just an approximation. Therefore, the solution of (B4) can still give a non-passive model (an example was given in Section 4). No similar assumptions are done with convex formulations [23] as well as the PFVF approach proposed in this paper.

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